USEMO
Ersatz Math Olympiad

Language: **English**

Day: **1**

Saturday, October 25, 2025

Problem 1. Find all real numbers λ for which there exists an integer $n \geq 2$ and an arithmetic progression $(a_0, a_1, \dots, a_n)$ of real numbers (in that order) such that the identity

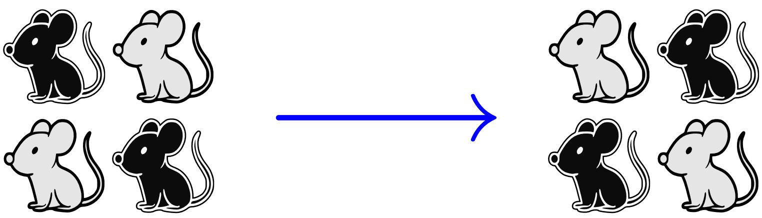
$$(X-\lambda)(X-\lambda^2)\dots(X-\lambda^n) = a_0X^n + a_1X^{n-1} + a_2X^{n-2} + \dots + a_n$$

is true for every real number X.

Problem 2. Let ABC be a fixed triangle with circumcircle ω. Consider P a variable point inside ABC. Ray BP meets side AC at Y while ray CP meets side AB at X. Let Q be the second intersection of ω and the circumcircle of triangle AXY. Let K be the second intersection of ray AP and ω.

Prove that as P varies, the circumcircles of triangle QPK all have a common radical center.

Problem 3. Suppose 2025 black and white mice are arranged in a 45×45 grid. A set of four mice is *special* if the four mice form a contiguous 2×2 square, the top-left mouse and the bottom-right mouse are both black, and the bottom-left mouse and top-right mouse are both white. A *move* swaps the positions of the black and white mice in a special set, as shown below.



Across all possible initial configurations of mice, what is the maximum number of moves that one could make?

The organizers of the USEMO are grateful to be sponsored this year by the CoRe Lab, Institute of Artificial Intelligence, Peking University.

Time limit: 4 hours and 30 minutes.
Each problem is worth seven points.

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Language: **English**

Day: **2**

Sunday, October 26, 2025

Problem 4. Determine all odd integers $n \geq 3$ with the following property: Let S denote the set of all positive integers less than n which are relatively prime to n, and let $k = \frac{1}{2}|S|$. Then one can label the integers in S by $a_1, \dots, a_{2k}$ in some order such that

$$\sum_{i=1}^{k} a_i^2 = \frac{1}{2}\sum_{i=1}^{k} a_i a_{i+k}.$$

Problem 5. Azza and Bob are playing the *squeakuences game*, a game whose rules depend on two positive integers n and g known to both players. A *squeakuence* is an ordered sequence of 100 integers (not necessarily positive). At the start of the game, Azza gives Bob a list of n different squeakuences, and Bob secretly picks one squeakuence and copies it into a notebook which Azza cannot see.

On each turn of the game, Azza makes up to g guesses for the squeakuence currently in the notebook. Bob hears all of Azza's guesses. If any of Azza's guesses are correct, the game ends and Azza wins. Otherwise, Bob privately chooses an index $1 \leq i \leq 100$ and an integer $\delta \in \{-1, 0, 1\}$, and secretly modifies the squeakuence written in the notebook by adding δ to its ith entry. (Azza is not told either i or δ. The values of i and δ can change from turn to turn.)

Find the smallest real number α for which there exists $C > 0$ making the following statement true: Azza can always guarantee winning the squeakuences game provided that $g > Cn^{\alpha}$.

Problem 6. Let k be a positive integer not divisible by 6. Suppose that there exists a prime p such that p divides both $2025^k - 1$ and $2026^k - 1$. Prove that $p < 3^k$.

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