OTIS Mock AIME I 2026

https://web.evanchen.cc/mockaime.html

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Instructions

1. This is a 15-question, 3-hour examination. All answers are integers ranging from 000 to 999, inclusive. Your score will be the number of correct answers; i.e., there is neither partial credit nor a penalty for wrong answers.

2. No aids other than writing utensils and erasers, scratch paper, graph paper, ruler, compass, protractor are permitted. In particular, books, notes, calculators, cell phones, computers, abacuses, ChatGPT, magic crystal balls, etc., are all prohibited.

3. Problem statements follow ARML conventions, so for example $\lfloor x \rfloor$ is the floor of x. You may refer to these conventions during the exam.

4. You may submit your answers at https://forms.gle/vbZWobCYUgwcq4t98 up until the deadline of 20 January 2026 23:59 Pacific time. Shortly after that answers and solutions will be posted on Evan's website. Please avoid public discussion of problems before that date. (Private discussions with people who have finished or do not plan to submit is OK.)

This is the OTIS Mock AIME, so it'll feel harder than an actual AIME. Stay determined and good luck!

Problems

Problem 1. Compute the smallest three-digit positive integer n such that the decimal digits of $3n$ and $4n$ are permutations of each other.

Problem 2. Compute the sum of all positive integers n for which $\operatorname{lcm}(1,\dots,n)$ can be written as the product of 10 distinct pairwise coprime positive integers less than or equal to n.

Problem 3. A triangle with area 90 is inscribed in a circle of radius 50. Compute the product of the lengths of the three altitudes of the triangle.

Problem 4. Compute the number of rectangles $\mathcal{R}$ that can be drawn in the 8×8 grid below such that

- the edges of $\mathcal{R}$ lie along the gridlines;
- at least one cell of $\mathcal{R}$ is labeled with a multiple of 9.

For example, one such rectangle would be formed by taking the cells labeled 9, 10, 11, 17, 18, 19.

0	1	2	3	4	5	6	7
8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23
24	25	26	27	28	29	30	31
32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

Problem 5. Triangle ABC has $AB = 65$, $BC = 70$, $CA = 75$. Let $\mathcal{P}$ be a parabola with focus A and directrix BC, and let $\mathcal{P}$ intersect segments AB and AC at D and E respectively. Compute the area of triangle ADE.

Problem 6. Let k, p, q be positive integers such that $\gcd(pq, 3) = 1$ and

$$\sum_{\substack{a+b+c=81 \\ a,b,c\ge 0}} \frac{1}{a!b!c!} = \frac{3^k p}{q}.$$

Compute k. Here the summation is over all triples (a, b, c) of nonnegative integers with sum 81.

Problem 7. A pair of nonnegative integers (i, u) is said to be compatible if $i < 3u$ and $u < 3i$. Let N be the number of compatible pairs of integers (i, u) for $0 \le i, u < 300$. Compute the remainder when N is divided by 1000.

Problem 8. Let p and q be relatively prime integers (not necessarily positive) satisfying

$$\frac{p}{q} = \cos^3(20^\circ)\cos^3(140^\circ) + \cos^3(140^\circ)\cos^3(260^\circ) + \cos^3(260^\circ)\cos^3(20^\circ).$$

Compute $p^2 + q^2$.

Problem 9. A function f taking pairs of nonnegative integers to nonnegative integers satisfies $f(0,0) = 0$ and

$$f(x,y) = f(y,x) = f(x+y,y) \text{ and } f(x,x) = f(x^2-1,x^3-1)+x$$

for all integers $x \ge 1$ and $y \ge 0$. Compute the remainder when

$$f(1,100)+f(2,100)+\cdots+f(100,100)$$

is divided by 1000.

Problem 10. Quadrilateral $ABCD$ with area k satisfies $\angle B = \angle C = \angle D$. Suppose $BC = 29$, $AD = 41$, and the distance from A to CD is 40. Compute $\lfloor k \rfloor$.

Problem 11. Let $e \approx 2.718$ denote the base of the natural logarithm ln. Compute the remainder when

$$\frac{1}{e}\sum_{i\ge1}\sum_{j\ge1}\sum_{k=1}^{2026}\frac{\binom{i-1}{j-1}\ln(k)^j}{j(i-1)!}$$

is divided by 1000.

Problem 12. Oscar the otter is reading the seven volumes of his favorite book series, Harry Otter. He has them all in a row in his bookshelf, but he wants to put them in chronological order. He wishes to do this by making *swaps*, where in one swap he will switch the positions of any two books in the row (these two books do *not* necessarily have to be adjacent). If the books in the row are in a random permutation of the chronological ordering, then the expected number of swaps Oscar needs for the books in the row to be in chronological order is $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m+n$.

Problem 13. Let $ABCD$ be a cyclic quadrilateral whose perimeter is 2048 and area is $2026(1+\sqrt{2})$. Assume $\angle C = 45^\circ$. Given that the lengths BC and DC are both integers, compute $\min(BC, DC)$.

Problem 14. Let N be the number of ways to label the vertices of a regular 13-gon, each with an integer from 1 through 14 (repetitions allowed), so that no set of consecutive labels has a sum divisible by 15. Compute the remainder when N is divided by 1000.

Problem 15. Let a be the smallest positive integer such that $a^2 - 2^{15}$ is divisible by 127^4. Compute the remainder when a is divided by 1000.