JMO 2026 Solution Notes

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This is a compilation of solutions for the 2026 JMO. The ideas of the solution are a mix of my own work, the solutions provided by the competition organizers, and solutions found by the community. However, all the writing is maintained by me.

These notes will tend to be a bit more advanced and terse than the "official" solutions from the organizers. In particular, if a theorem or technique is not known to beginners but is still considered "standard", then I often prefer to use this theory anyways, rather than try to work around or conceal it. For example, in geometry problems I typically use directed angles without further comment, rather than awkwardly work around configuration issues. Similarly, sentences like "let $\mathbb{R}$ denote the set of real numbers" are typically omitted entirely.

Corrections and comments are welcome!

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§0 Problems

1. Let a, b, c be distinct positive integers such that $ab+c=c^2$. Prove that $(a-b)^2 \geq 4c$.

2. There are m foxes and n bunnies sitting in $m+n$ seats around a circular table, where all animals are distinguishable. If a fox and a bunny are sitting next to each other, they may swap positions with each other. For a fixed starting configuration, determine the number of configurations that can be reached by a sequence of swaps. (Rotations and reflections of a configuration are considered distinct.)

3. Let ABC be an acute scalene triangle with no angle equal to 60°. Let ω be the circumcircle of ABC. Let Δ_B be the equilateral triangle with three vertices on ω, one of which is B. Let ℓ_B be the line through the two vertices of Δ_B other than B. Let Δ_C and ℓ_C be defined analogously. Let Y be the intersection of AC and ℓ_B, and let Z be the intersection of AB and ℓ_C.

   Suppose that the circumcircle of AYZ intersects ω at $P \neq A$, BC intersects YZ at D, and PA intersects YZ at E. Prove that $PE = PD$.

4. Triangle ABC has circumcircle ω and circumcenter O. Lines AO and BC meet at point D. Let X be the A-excenter of $\triangle ABD$ and let Y be the A-excenter of $\triangle ACD$. Prove that if X lies on ω, then Y also lies on ω.

5. A positive integer n is called *solitary* if, for any non-negative integers a and b such that $a+b=n$, either a or b contains the digit "1". Determine, with proof, the number of solitary integers less than 10^{2026}.

6. Emily has a red sheet of paper. She draws 2026 circles (not necessarily of equal size) on the piece of paper. She chooses a circle to color black, then cuts the paper around the circumference of all 2026 circles. She then separates the pieces of paper, into at least 2 black pieces and some number of red pieces. Is it possible that all black pieces are congruent?

§1 Solutions to Day 1

§1.1 JMO 2026/1, proposed by Milan Haiman

Available online at https://aops.com/community/p37578110.
Video at https://youtu.be/cvsXd6eFn-g.

Problem statement

Let a, b, c be distinct positive integers such that $ab+c=c^2$. Prove that $(a-b)^2 \ge 4c$.

Note that the conclusion is equivalent to

$$(a-b)^2 \ge 4c \iff (a+b)^2 \ge 4c + 4ab = 4c^2 \iff a+b \ge 2c.$$

We now solve the problem:

Claim — We have $a+b \ge 2c$.

Proof. Because of distinctness, we may as well assume WLOG that

$$a > c > c-1 > b.$$

Since the function

$$\begin{aligned} f \colon (\sqrt{c(c-1)}, \infty) &\to \mathbb{R} \\ x &\mapsto x + \frac{c(c-1)}{x} \end{aligned}$$

is strictly increasing, it follows that

$$a+b = f(a) > f(c) = 2c-1 \implies a+b \ge 2c.$$ □

§1.2 JMO 2026/2, proposed by Milan Haiman

🔗 *Available online at* https://aops.com/community/p37578101.
📽 *Video at* https://youtu.be/j12Ar6OP3n4.

Problem statement

There are m foxes and n bunnies sitting in $m+n$ seats around a circular table, where all animals are distinguishable. If a fox and a bunny are sitting next to each other, they may swap positions with each other. For a fixed starting configuration, determine the number of configurations that can be reached by a sequence of swaps. (Rotations and reflections of a configuration are considered distinct.)

To be written later.

§1.3 JMO 2026/3, proposed by Rogelio Guerrero

🔗 *Available online at* https://aops.com/community/p37578105.
📽 *Video at* https://youtu.be/fGN283EHQaU.

Problem statement

Let ABC be an acute scalene triangle with no angle equal to $60°$. Let ω be the circumcircle of ABC. Let Δ_B be the equilateral triangle with three vertices on ω, one of which is B. Let ℓ_B be the line through the two vertices of Δ_B other than B. Let Δ_C and ℓ_C be defined analogously. Let Y be the intersection of AC and ℓ_B, and let Z be the intersection of AB and ℓ_C.

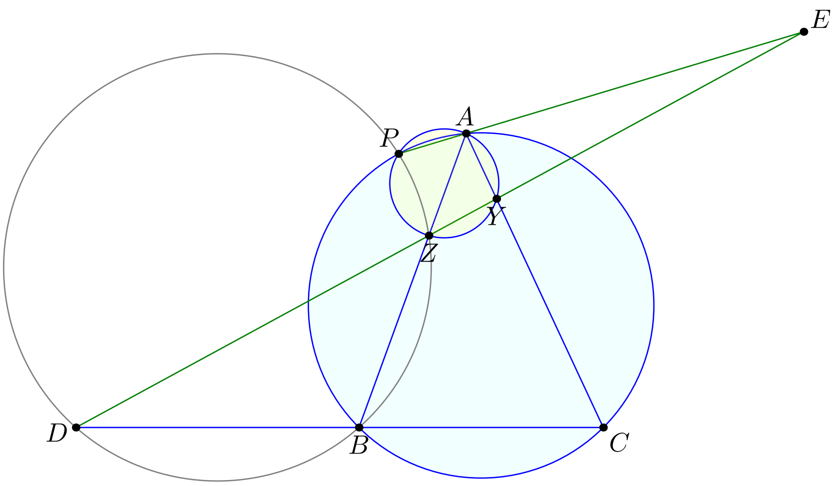
Suppose that the circumcircle of AYZ intersects ω at $P \neq A$, BC intersects YZ at D, and PA intersects YZ at E. Prove that $PE = PD$.

Because P is the Miquel point of $BZYC$, it follows $DBZP$ is cyclic. Hence

$$\measuredangle PDE = \measuredangle PDZ = \measuredangle PBZ = \measuredangle PBA$$
$$\measuredangle DEP = \measuredangle(\overline{YZ}, \overline{AP}).$$

So the problem is solved if we can show

$$\measuredangle PBA = \measuredangle(\overline{YZ}, \overline{AP}). \qquad (\bigstar)$$



We use complex numbers to prove $(\bigstar)$. Let $\omega = e^{2\pi i/3}$. Since Y is the intersection of a, c, ωb and $\omega^2 b$, we have

$$y = \frac{b^2(a+c) - ac(\omega b + \omega^2 b)}{b^2 - ac} = \frac{b \cdot (ab+bc+ca)}{b^2 - ac}.$$
$$z = \frac{c \cdot (ab+bc+ca)}{c^2 - ab}.$$

We compute the point P now.

Claim — We have
$$p = \frac{ab+bc+ca}{a+b+c}.$$

Proof. Note that
$$\frac{p-z}{p-y} = \frac{p-b}{p-c} \iff p = \frac{by-cz}{b+y-c-z}.$$
The numerator and denominator are respectively
$$\begin{aligned} by-cz &= (ab+bc+ca)\cdot\left(\frac{b^2}{b^2-ac} - \frac{c^2}{c^2-ab}\right) \\ &= (ab+bc+ca)\cdot\frac{a(c^3-b^3)}{(b^2-ac)(c^2-ab)} \\ &= -(ab+bc+ca)\cdot\frac{a(b-c)(b^2+bc+c^2)}{(b^2-ac)(c^2-ab)} \\ b+y-c-z &= \frac{(b-c)[(b^2-ac)(c^2-ab)-(ab+bc+ca)^2]}{(b^2-ac)(c^2-ab)} \\ &= \frac{(b-c)[-ab^3-ac^3+a^2bc-a^2b^2-c^2a^2-2abc(a+b+c)]}{(b^2-ac)(c^2-ab)} \\ &= \frac{-a(b-c)[a(b^2+bc+c^2)+b^3+c^3+2bc(b+c)]}{(b^2-ac)(c^2-ab)} \\ &= \frac{-a(b-c)(a+b+c)(b^2+bc+c^2)}{(b^2-ac)(c^2-ab)}. \end{aligned}$$

Dividing gives the conclusion. □

The desired conclusion (★) is encoded as
$$\mathbb{R} \ni \frac{p}{p+a} \div \frac{y-z}{p-a} = \frac{p-a}{p+a}\cdot\frac{p}{y-z} \iff \frac{p}{y-z} \in i\mathbb{R}$$
since $\frac{p-a}{p+a}$ is obviously pure imaginary (for $|a| = |p| = 1$). We compute
$$\begin{aligned} y-z &= (ab+bc+ca)\cdot\frac{b(c^2-ab)-c(b^2-ac)}{(b^2-ac)(c^2-ab)} \\ &= (ab+bc+ca)\cdot\frac{-(b-c)(bc+ab+ac)}{(b^2-ac)(c^2-ab)} \\ &= -(ab+bc+ca)^2\cdot\frac{b-c}{(b^2-ac)(c^2-ab)} \end{aligned}$$

Hence
$$\frac{p}{y-z} = \frac{(b^2-ac)(c^2-ab)}{(b-c)(a+b+c)(ab+bc+ca)}.$$
The conjugate is
$$\overline{\left(\frac{p}{y-z}\right)} = \frac{\left(\frac{1}{b^2}-\frac{1}{ac}\right)\left(\frac{1}{c^2}-\frac{1}{ab}\right)}{\left(\frac{1}{b}-\frac{1}{c}\right)\cdot\frac{ab+bc+ca}{abc}\cdot\frac{a+b+c}{abc}} = -\frac{p}{y-z}$$
as desired.

§2 Solutions to Day 2

§2.1 JMO 2026/4, proposed by Carl Schildkraut

Available online at https://aops.com/community/p37586265.
Video at https://youtu.be/dhI18BfnHUs.

Problem statement

Triangle ABC has circumcircle ω and circumcenter O. Lines AO and BC meet at point D. Let X be the A-excenter of $\triangle ABD$ and let Y be the A-excenter of $\triangle ACD$. Prove that if X lies on ω, then Y also lies on ω.

To be added. (Angle chasing, but super annoying.)

§2.2 JMO 2026/5, proposed by Carl Schildkraut

Available online at https://aops.com/community/p37586238.
Video at https://youtu.be/0gbE7VKeYEs.

Problem statement

A positive integer n is called *solitary* if, for any non-negative integers a and b such that $a+b=n$, either a or b contains the digit "1". Determine, with proof, the number of solitary integers less than 10^{2026}.

We claim that a number is solitary if and only if all the following hold:

- The digit 1 appears exactly once;
- Every digit (possibly none) to the left of the 1 is 0 or 2;
- Every digit (possibly none) to the right of the 1 is 9.

For example, 202201999999 is solitary.

¶ Proof all such numbers are solitary. The basic idea is to use induction as follows: If the last digit of n is 9, then the last digits of a and b sum to 9; hence we can ignore the last digit altogether. Thus, we reduce to the case where the last digit is 1.

We continue the induction in a similar way in this situation. Zero-pad all the numbers to be the same length as n. Take the leading 2 of n.

- If either or a and b has a leading digit 2, we can delete it and continue the induction.
- Otherwise, clearly one of a or b must have leading digit 1, as needed.

¶ Proof that every solitary number is of this form. Let n be solitary. In all the diagrams that follow, ellipses denote groups of digits other than 1 (possibly none).

We first reduce to the case where n has exactly one 1:

- By taking $b=0$, we see there is at least a single 1.
- Suppose n has an even number of 1's. The idea is to pair the 1's using blocks like $9\dots9$, as in the following diagram:

$$\begin{array}{rccccc} n= & \dots1 & \dots1 & \dots1 & \dots1 & \dots \\ a= & 0000 & 9993 & 0000 & 9993 & 0000 \\ b= & \dots0 & \dots8 & 0000 & \dots8 & \dots \end{array}$$

- Next, suppose n has an odd number of 1's and at least three 1's. The strategy is similar:

$$\begin{array}{rcccccc} n= & \dots1 & \dots1 & \dots1 & \dots1 & \dots1 & \dots \\ a= & 0000 & 9993 & 9993 & 0000 & 9993 & 0000 \\ b= & \dots1 & \dots7 & \dots8 & 0000 & \dots8 & \dots \end{array}$$

So, assume n has exactly one 1 appear.

- If a digit $b \neq 0, 9$ appears to the right of 1, let $c = b + 1$ to get

$$\begin{array}{rcccc} n = & \dots & 1 & \dots b & \dots \\ a = & 0000 & 0 & 9999 & 0000 \\ b = & \dots & 0 & \dots c & \dots \end{array}$$

  If $b = 0$, one can instead replace 9999 with 9998 and choose $c = 2$.

- Finally, if a digit $d \neq 0, 2$ appears to the left of 1, let $e = d - 1$ and use a bunch of 9's:

$$\begin{array}{rcccc} n = & \dots d & \dots & 1 & \dots \\ a = & 0000 & 9999 & 9 & 0000 \\ a = & \dots e & \dots & 2 & \dots \end{array}$$

¶ Final count. Zero-pad the number so that it has exactly 2026 digits. If we pick the digit 1 to appear at the i'th place from the left, for $1 \leq i \leq 2026$, then there are 2^{i-1} ways to pick the first $i - 1$ digits (either 0 or 2); the rest are all 9. Hence, the answer is

$$\sum_{i=1}^{2026} 2^{i-1} = \boxed{2^{2026} - 1}.$$

§2.3 JMO 2026/6, proposed by Carl Schildkraut

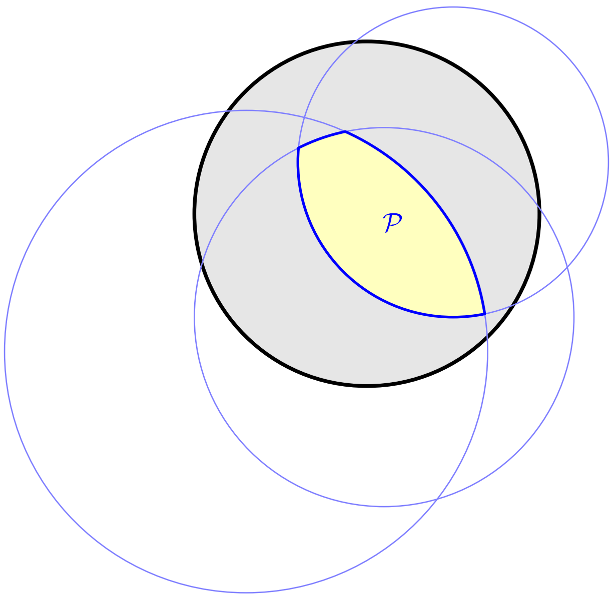
Available online at https://aops.com/community/p37586342.
Video at https://youtu.be/LZ4J5GQwrgA.

Problem statement

Emily has a red sheet of paper. She draws 2026 circles (not necessarily of equal size) on the piece of paper. She chooses a circle to color black, then cuts the paper around the circumference of all 2026 circles. She then separates the pieces of paper, into at least 2 black pieces and some number of red pieces. Is it possible that all black pieces are congruent?

No, this is impossible.

Let Γ be the black circle, and for each point inside it consider the number of the 2026 circles it's contained in. Let X be a point for which that number achieves its maximal value, and consider the piece $\mathcal{P}$ containing X.



Claim — All the curves that bound $\mathcal{P}$ point outwards.

Proof. If not, then one could replace X with a point Y just across an inwards-pacing curve bounding $\mathcal{P}$. □

However, it's clear that if there are more than two black pieces, then some piece has a curve that points inwards. So it's impossible that the pieces are all congruent.