Team Selection Test for the 56$^{\text{th}}$ International Mathematical Olympiad

United States of America

Day I

Thursday, December 11, 2014

Time limit: 4.5 hours. Each problem is worth 7 points.

IMO TST 1. Let ABC be a scalene triangle with incenter I whose incircle is tangent to $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ at D, E, F, respectively. Denote by M the midpoint of $\overline{BC}$ and let P be a point in the interior of $\triangle ABC$ so that $MD = MP$ and $\angle PAB = \angle PAC$. Let Q be a point on the incircle such that $\angle AQD = 90°$. Prove that either $\angle PQE = 90°$ or $\angle PQF = 90°$.

IMO TST 2. Prove that for every positive integer n, there exists a set S of n positive integers such that for any two distinct $a, b \in S$, $a - b$ divides a and b but none of the other elements of S.

IMO TST 3. A physicist encounters 2015 atoms called usamons. Each usamon either has one electron or zero electrons, and the physicist can't tell the difference. The physicist's only tool is a diode. The physicist may connect the diode from any usamon A to any other usamon B. (This connection is directed.) When she does so, if usamon A has an electron and usamon B does not, then the electron jumps from A to B. In any other case, nothing happens. In addition, the physicist cannot tell whether an electron jumps during any given step. The physicist's goal is to isolate two usamons that she is 100% sure are currently in the same state. Is there any series of diode usage that makes this possible?

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Day II

Thursday, January 22, 2015

Time limit: 4.5 hours. Each problem is worth 7 points.

IMO TST 4. Let $f\colon \mathbb{Q} \to \mathbb{Q}$ be a function such that for any $x, y \in \mathbb{Q}$, the number $f(x+y) - f(x) - f(y)$ is an integer. Decide whether there must exist a constant c such that $f(x) - cx$ is an integer for every rational number x.

IMO TST 5. Fix a positive integer n. A tournament on n vertices has all its edges colored by χ colors, so that any two directed edges $u \to v$ and $v \to w$ have different colors. Over all possible tournaments on n vertices, determine the minimum possible value of χ.

IMO TST 6. Let ABC be a non-equilateral triangle and let M_a, M_b, M_c be the midpoints of the sides BC, CA, AB, respectively. Let S be a point lying on the Euler line. Denote by X, Y, Z the second intersections of M_aS, M_bS, M_cS with the nine-point circle. Prove that AX, BY, CZ are concurrent.