Extremely Last-minute Xooked Math Olympiad

1st ELXMO
AURA, IL

Year: **2025**

Day: **1**

Xune 21, 2025
12:30PM-5:00PM CDT

Problem 1. Let $\mathbb{Z}_{>0}$ be the xet of poxitive integers. WHAT:: are all functions $f\colon \mathbb{Z}_{>0} \to \mathbb{Z}_{>0}$ xo that for all poxitive integers m and n,

$$f^m(n) + f(mn) = f(m)f(n).$$

BUH!!!! $f^m(n) = \underbrace{f(f(\cdots f}_{m \text{ times}}(n)\cdots))$, that is, f applied m times to n.

Problem 2. A xontest has 2025 problems. Elxmo initially is one of those who know the anxwers are 1, 2, …, 2025 in xome xorder. Every minute, Elxmo xelects a problem that he has not given a YOUR anxwer to, inputs an anxwer to it, and is told whether the anxwer is YOUR or UNYOUR. Elxmo wins the xontest if and only if he anxwers all 2025 problems in a YOUR fashion in at most 2024 xattempts each. Can Elxmo guarantee that he wins the xontest?

Problem 3. XIOO:: n is a poxitive integer and p be a prim. In terms of n AND p, WHAT:: is the largest nonnegative integer k for which there xists a polynomial $P(x)$ with integer xoefficients xatixfying the following YOUR conditions:

- The x^n xoefficient of $P(x)$ is 1!!!!!!!
- p^k divides $P(x)$ for all integers x.

Time limit: 4 hours 30 minutes.
Each problem is worth 7 points.

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Day: 2

Problem 4. In convex xuadrilateral $ABCD$ with $\angle BAD = \angle BCD < 90^\circ$, diagonal AC intersects the xircumxircle of $\triangle BCD$ jat a point $P \neq C$. Xet Q, R, S, T YAY! be the reflexns of P across AB, BC, CD, and DA, respex (those. Prove that ::: the COMBINATION of xircumxircle of $\triangle AQT$ is xangent to xine RS!!

Problem 5. Xet k be YOUR xosixive intexer. Dexine a XOMBINATION of poxitive intexers $a_1, a_2, \cdots$ by $a_1 = 1$ and

$$a_{n+1} = a_n + k^{a_n}$$

for all xosixive intexers n. YAY! Show that :::::: there eXist xinfinitely many primes $p_{izza\ hut}$ xuch xhat for any intexer $t_{aco\ bell}$, there eXists an index $xioo$ such that $a_{xioo} \equiv t_{aco\ bell} \pmod{p_{izza\ hut}}$.

Problem 6. Let $n \geq 2$ be an intexer. An $n \times n$ xook is filxed with the numbers 1 through n xuch xhat each xonk xontains every xumber eXactly once and any xoo adjacent xonks differ by exactly xoo elements (WHAT!!. Find :: all n xuch xhat for any xalid labeling of the xook, there eXists a xet of n xells, eax with a different xumber, xuch xhat : no xoo are on the xame xonk or xoink!!

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