Elsmo eLsmo elSmo elsMo elsmO

23rd ELSMO

Pittsburgh pIttsburgh piTtsburgh pitTsburgh pittSburgh pittsBurgh pittsbUrgh pittsbuRgh pittsburGh pittsburgH, Pa pA

Year yEar yeAr yeaR: **2023**

Day dAy daY: **1**

Saturday sAturday saTurday satUrday satuRday saturDay saturdAy saturdaY, June jUne juNe junE Tenth tEnth teNth tenTh tentH, Twenty tWenty twEnty tweNty twenTy twentY Twenty-three tWenty-three twEnty-three tweNty-three twenTy-three twentY-three twenty-Three twenty-tHree twenty-thRee twenty-thrEe twenty-threE One-twenty oNe-twenty onE-twenty one-Twenty one-tWenty one-twEnty one-tweNty one-twenTy one-twentY To tO Five-fifty fIve-fifty fiVe-fifty fivE-fifty five-Fifty five-fIfty five-fiFty five-fifTy five-fiftY Pm pM, Eastern eAstern eaStern easTern eastErn easteRn easterN Daylight dAylight daYlight dayLight daylIght dayliGht dayligHt daylighT Time tIme tiMe timE

Problem pRoblem prOblem proBlem probLem problEm probleM 1. Let lEt leT M Be bE A Positive pOsitive poSitive posItive posiTive positIve positiVe positivE Integer iNteger inTeger intEger inteGer integEr integeR. Find fInd fiNd finD, In iN Terms tErms teRms terMs termS Of oF M, All aLl alL Polynomials pOlynomials poLynomials polYnomials polyNomials polynOmials polynoMials polynomIals polynomiAls polynomiaLs polynomialS $P(x)$ $p(X)$ With wIth wiTh witH Integer iNteger inTeger intEger inteGer integEr integeR Coefficients cOefficients coEfficients coeFficients coefFicients coeffIcients coeffiCients coefficIents coefficiEnts coefficieNts coefficienTs coefficientS Such sUch suCh sucH That tHat thAt thaT For fOr foR Every eVery evEry eveRy everY Integer iNteger inTeger intEger inteGer integEr integeR N, There tHere thEre theRe therE Exists eXists exIsts exiSts exisTs existS An aN Integer iNteger inTeger intEger inteGer integEr integeR K Such sUch suCh sucH That tHat thAt thaT $P(k) = n^m$ $p(K) = n^m$ $p(k) = N^m$ $p(k) = n^M$.

Problem pRoblem prOblem proBlem probLem problEm probleM 2. Let lEt leT A, B, And aNd anD N Be bE Positive pOsitive poSitive posItive posiTive positIve positiVe positivE Integers iNtegers inTegers intEgers inteGers integErs integeRs integerS. A Lemonade lEmonade leMonade lemOnade lemoNade lemonAde lemonaDe lemonadE Stand sTand stAnd staNd stanD Owns oWns owNs ownS N Cups cUps cuPs cupS, All aLl alL Of oF Which wHich whIch whiCh whicH Are aRe arE Initially iNitially inItially iniTially initIally initiAlly initiaLly initialLy initiallY Empty eMpty emPty empTy emptY. The tHe thE Lemonade lEmonade leMonade lemOnade lemoNade lemonAde lemonaDe lemonadE Stand sTand stAnd staNd stanD Has hAs haS A *Filling fIlling fiLling filLing fillIng filliNg fillinG Machine mAchine maChine macHine machIne machiNe machinE* And aNd anD An aN *Emptying eMptying emPtying empTying emptYing emptyIng emptyiNg emptyinG Machine mAchine maChine macHine machIne machiNe machinE*, Which wHich whIch whiCh whicH Operate oPerate opErate opeRate operAte operaTe operatE According aCcording acCording accOrding accoRding accorDing accordIng accordiNg accordinG To tO The tHe

Time tIme tiMe timE Limit lImit liMit limIt limiT: Four fOur foUr fouR Hours hOurs hoUrs houRs hourS Thirty tHirty thIrty thiRty thirTy thirtY Minutes mInutes miNutes minUtes minuTes minutEs minuteS .

Each eAch eaCh eacH Problem pRoblem prOblem proBlem probLem problEm probleM Is iS Worth wOrth woRth worTh wortH 7 Points pOints poInts poiNts poinTs pointS.

thE Following fOllowing foLlowing folLowing follOwing folloWing followIng followiNg followinG Rules rUles ruLes rulEs ruleS:

- If iF At aT Any aNy anY Moment mOment moMent momEnt momeNt momenT, *A* Completely cOmpletely coMpletely comPletely compLetely complEtely compleTely completEly completeLy completelY Empty eMpty emPty empTy emptY Cups cUps cuPs cupS Are aRe arE Available aVailable avAilable avaIlable avaiLable availAble availaBle availabLe availablE, The tHe thE Filling fIlling fiLling filLing fillIng filliNg fillinG Machine mAchine maChine macHine machIne machiNe machinE Spends sPends spEnds speNds spenDs spendS The tHe thE Next nExt neXt nexT *A* Minutes mInutes miNutes minUtes minuTes minutEs minuteS Filling fIlling fiLling filLing fillIng filliNg fillinG Those tHose thOse thoSe thosE *A* Cups cUps cuPs cupS Simultaneously sImultaneously siMultaneously simUltaneously simuLtaneously simulTaneously simultAneously simultaNeously simultanEously simultaneOusly simultaneoUsly simultaneouSly simultaneousLy simultaneouslY And aNd anD Doing dOing doIng doiNg doinG Nothing nOthing noThing notHing nothIng nothiNg nothinG Else eLse elSe elsE.
- If iF At aT Any aNy anY Moment mOment moMent momEnt momeNt momenT, *B* Completely cOmpletely coMpletely comPletely compLetely complEtely compleTely completEly completeLy completelY Full fUll fuLl fulL Cups cUps cuPs cupS Are aRe arE Available aVailable avAilable avaIlable avaiLable availAble availaBle availabLe availablE, The tHe thE Emptying eMptying emPtying empTying emptYing emptyIng emptyiNg emptyinG Machine mAchine maChine macHine machIne machiNe machinE Spends sPends spEnds speNds spenDs spendS The tHe thE Next nExt neXt nexT *B* Minutes mInutes miNutes minUtes minuTes minutEs minuteS Emptying eMptying emPtying empTying emptYing emptyIng emptyiNg emptyinG Those tHose thOse thoSe thosE *B* Cups cUps cuPs cupS Simultaneously sImultaneously siMultaneously simUltaneously simuLtaneously simulTaneously simultAneously simultaNeously simultanEously simultaneOusly simultaneoUsly simultaneouSly simultaneousLy simultaneouslY And aNd anD Doing dOing doIng doiNg doinG Nothing nOthing noThing notHing nothIng nothiNg nothinG Else eLse elSe elsE.

Suppose sUppose suPpose supPose suppOse suppoSe supposE That tHat thAt thaT After aFter afTer aftEr afteR A Sufficiently sUfficiently suFficiently sufFiciently suffIciently suffiCiently sufficIently sufficiEntly sufficieNtly sufficienTly sufficientLy sufficientlY Long lOng loNg lonG Time tIme tiMe timE Has hAs haS Passed pAssed paSsed pasSed passEd passeD, Both bOth boTh botH The tHe thE Filling fIlling fiLling filLing fillIng filliNg fillinG Machine mAchine maChine macHine machIne machiNe machinE And aNd anD Emptying eMptying emPtying empTying emptYing emptyIng emptyiNg emptyinG Machine mAchine

Time tIme tiMe timE Limit lImit liMit limIt limiT: Four fOur foUr fouR Hours hOurs hoUrs houRs hourS Thirty tHirty thIrty thiRty thirTy thirtY Minutes mInutes miNutes minUtes minuTes minutEs minuteS .
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maChine macHine machIne machiNe machinE Work wOrk woRk worK Without wIthout wiThout witHout withOut withoUt withouT Pausing pAusing paUsing pauSing pausIng pausiNg pausinG. Find fInd fiNd finD, In iN Terms tErms teRms terMs termS Of oF A And aNd anD B, The tHe thE Least lEast leAst leaSt leasT Possible pOssible poSsible posSible possIble possiBle possibLe possiblE Value vAlue vaLue valUe valuE Of oF N.

Problem pRoblem prOblem proBlem probLem problEm probleM 3. Convex cOnvex coNvex conVex convEx conveX Quadrilaterals qUadrilaterals quAdrilaterals quaDrilaterals quadRilaterals quadrIlaterals quadriLaterals quadrilAterals quadrilaTerals quadrilatErals quadrilateRals quadrilaterAls quadrilateraLs quadrilateralS $Abcd$ $aBcd$ $abCd$ $abcD$, $A_1b_1c_1d_1$ $a_1B_1c_1d_1$ $a_1b_1C_1d_1$ $a_1b_1c_1D_1$, And aNd anD $A_2b_2c_2d_2$ $a_2B_2c_2d_2$ $a_2b_2C_2d_2$ $a_2b_2c_2D_2$ Are aRe arE Similar sImilar siMilar simIlar simiLar similAr similaR With wIth wiTh witH Vertices vErtices veRtices verTices vertIces vertiCes verticEs verticeS In iN Order oRder orDer ordEr ordeR. Points pOints poInts poiNts poinTs pointS A, A_1, B_2, B Are aRe arE Collinear cOllinear coLlinear colLinear collInear colliNear collinEar collineAr collineaR In iN Order oRder orDer ordEr ordeR, Points pOints poInts poiNts poinTs pointS B, B_1, C_2, C Are aRe arE Collinear cOllinear coLlinear colLinear collInear colliNear collinEar collineAr collineaR In iN Order oRder orDer ordEr ordeR, Points pOints poInts poiNts poinTs pointS C, C_1, D_2, D Are aRe arE Collinear cOllinear coLlinear colLinear collInear colliNear collinEar collineAr collineaR In iN Order oRder orDer ordEr ordeR, And aNd anD Points pOints poInts poiNts poinTs pointS D, D_1, A_2, A Are aRe arE Collinear cOllinear coLlinear colLinear collInear colliNear collinEar collineAr collineaR In iN Order oRder orDer ordEr ordeR. Diagonals dIagonals diAgonals diaGonals diagOnals diagoNals diagonAls diagonaLs diagonalS Ac aC And aNd anD Bd bD Intersect iNtersect inTersect intErsect inteRsect interSect intersEct interseCt intersecT At aT P, Diagonals dIagonals diAgonals diaGonals diagOnals diagoNals diagonAls diagonaLs diagonalS A_1c_1 a_1C_1 And aNd anD B_1d_1 b_1D_1 Intersect iNtersect inTersect intErsect inteRsect interSect intersEct interseCt intersecT At aT P_1, And aNd anD Diagonals dIagonals diAgonals diaGonals diagOnals diagoNals diagonAls diagonaLs diagonalS A_2c_2 a_2C_2 And aNd anD B_2d_2 b_2D_2 Intersect iNtersect inTersect intErsect inteRsect interSect intersEct interseCt intersecT At aT P_2. Prove pRove prOve proVe provE That tHat thAt thaT Points pOints poInts poiNts poinTs pointS P, P_1, And aNd anD P_2 Are aRe arE Collinear cOllinear coLlinear colLinear collInear colliNear collinEar collineAr collineaR.

Time tIme tiMe timE Limit lImit liMit limIt limiT: Four fOur foUr fouR Hours hOurs hoUrs houRs hourS Thirty tHirty thIrty thiRty thirTy thirtY Minutes mInutes miNutes minUtes minuTes minutEs minuteS .

Each eAch eaCh eacH Problem pRoblem prOblem proBlem probLem problEm probleM Is iS Worth wOrth woRth worTh wortH 7 Points pOints poInts poiNts poinTs pointS.

Sunday sUnday suNday sunDay sundAy sundaY, June jUne juNe junE Eighteenth eIghteenth eiGhteenth eigHteenth eighTeenth eightEenth eighteEnth eighteeNth eighteenTh eighteentH, Twenty tWenty twEnty tweNty twenTy twentY Twenty tWenty twEnty tweNty twenTy twentY Three tHree thRee thrEe threE One-twenty oNe-twenty onE-twenty one-Twenty one-tWenty one-twEnty one-tweNty one-twenTy one-twentY To tO Five-fifty fIve-fifty fiVe-fifty fivE-fifty five-Fifty five-fIfty five-fiFty five-fifTy five-fiftY Pm pM, Eastern eAstern eaStern easTern eastErn easteRn easterN Daylight dAylight daYlight dayLight daylIght dayliGht dayligHt daylighT Time tIme tiMe timE

On oN Day dAy daY One oNe onE, We wE Received rEceived reCeived recEived receIved receiVed receivEd receiveD Complaints cOmplaints coMplaints comPlaints compLaints complAints complaInts complaiNts complainTs complaintS That tHat thAt thaT The tHe thE Elsmo eLsmo elSmo elsMo elsmO Was wAs waS Too tOo toO Boring bOring boRing borIng boriNg borinG. To tO Fix fIx fiX This tHis thIs thiS, We wE Have hAve haVe havE Issued iSsued isSued issUed issuEd issueD A Message mEssage meSsage mesSage messAge messaGe messagE: I Lost lOst loSt losT The tHe thE Game gAme gaMe gamE.

Problem pRoblem prOblem proBlem probLem problEm probleM 1. Let lEt leT Abc aBc abC Be bE An aN Acute aCute acUte acuTe acutE Scalene sCalene scAlene scaLene scalEne scaleNe scalenE Triangle tRiangle trIangle triAngle triaNgle trianGle triangLe trianglE With wIth wiTh witH Orthocenter oRthocenter orThocenter ortHocenter orthOcenter orthoCenter orthocEnter orthoceNter orthocenTer orthocentEr orthocenteR H. Line lIne liNe linE Bh bH Intersects iNtersects inTersects intErsects inteRsects interSects intersEcts interseCts intersecTs intersectS $\overline{Ac}$ $\overline{aC}$ At aT E And aNd anD Line lIne liNe linE Ch cH Intersects iNtersects inTersects intErsects inteRsects interSects intersEcts interseCts intersecTs intersectS $\overline{Ab}$ $\overline{aB}$ At aT F. Let lEt leT X Be bE The tHe thE Foot fOot foOt fooT Of oF The tHe thE Perpendicular pErpendicular peRpendicular perPendicular perpEndicular perpeNdicular perpenDicular perpendIcular perpendiCular perpendicUlar perpendicuLar perpendiculAr perpendiculaR From fRom frOm froM H To tO The tHe thE Line lIne liNe linE Through tHrough thRough thrOugh throUgh throuGh througH A Parallel pArallel paRallel parAllel paraLlel paralLel parallEl paralleL To tO $\overline{Ef}$ $\overline{eF}$. Point pOint poInt poiNt poinT B_1 Lies lIes liEs lieS On oN Line lIne liNe linE Xf xF Such sUch suCh sucH That tHat thAt thaT $\overline{Bb_1}$ $\overline{bB_1}$ Is iS Parallel pArallel paRallel parAllel paraLlel paralLel parallEl paralleL To tO $\overline{Ac}$ $\overline{aC}$, And aNd anD Point pOint poInt poiNt poinT C_1 Lies lIes liEs lieS On oN Line lIne liNe linE Xe xE Such sUch suCh sucH That tHat thAt thaT $\overline{Cc_1}$ $\overline{cC_1}$ Is iS Parallel pArallel paRallel parAllel paraLlel paralLel parallEl paralleL To tO $\overline{Ab}$ $\overline{aB}$. Prove pRove prOve proVe provE That tHat thAt thaT

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Each eAch eaCh eacH Problem pRoblem prOblem proBlem probLem problEm probleM Is iS Worth wOrth woRth worTh wortH 7 Points pOints poInts poiNts poinTs pointS.

Points pOints poInts poiNts poinTs pointS B, C, B_1, C_1 Are aRe arE Concyclic cOncyclic coNcyclic conCyclic concYclic concyClic concycLic concyclIc concycliC.

Problem pRoblem prOblem proBlem probLem problEm probleM 2. Find fInd fiNd finD The tHe thE Least lEast leAst leaSt leasT Positive pOsitive poSitive posItive posiTive positIve positiVe positivE Integer iNteger inTeger intEger inteGer integEr integeR M For fOr foR Which wHich whIch whiCh whicH There tHere thEre theRe therE Exist eXist exIst exiSt exisT A Positive pOsitive poSitive posItive posiTive positIve positiVe positivE Integer iNteger inTeger intEger inteGer integEr integeR n And aNd anD Polynomials pOlynomials poLynomials polYnomials polyNomials polynOmials polynoMials polynomIals polynomiAls polynomiaLs polynomialS $P_1(x)$ $p_1(X)$, $P_2(x)$ $p_2(X)$, $\ldots$, $P_n(x)$ $p_n(X)$ With wIth wiTh witH Integer iNteger inTeger intEger inteGer integEr integeR Coefficients cOefficients coEfficients coeFficients coefFicients coeffIcients coeffiCients coefficIents coefficiEnts coefficieNts coefficienTs coefficientS Satisfying sAtisfying saTisfying satIsfying satiSfying satisFying satisfYing satisfyIng satisfyiNg satisfyinG $Mx = p_1(x)^3 + p_2(x)^3 + \cdots + p_n(x)^3$. $mX = p_1(x)^3 + p_2(x)^3 + \cdots + p_n(x)^3$. $mx = P_1(x)^3 + p_2(x)^3 + \cdots + p_n(x)^3$. $mx = p_1(X)^3 + p_2(x)^3 + \cdots + p_n(x)^3$. $mx = p_1(x)^3 + P_2(x)^3 + \cdots + p_n(x)^3$. $mx = p_1(x)^3 + p_2(X)^3 + \cdots + p_n(x)^3$. $mx = p_1(x)^3 + p_2(X)^3 + \circ \cdots + p_n(x)^3$. $mx = p_1(x)^3 + p_2(X)^3 + \cdot \circ \cdot + p_n(x)^3$. $mx = p_1(x)^3 + p_2(X)^3 + \cdot\cdot \circ + p_n(x)^3$. $mx = p_1(x)^3 + p_2(x)^3 + \cdots + P_n(x)^3$. $mx = p_1(x)^3 + p_2(x)^3 + \cdots + pN(x)^3$. $mx = p_1(x)^3 + p_2(x)^3 + \cdots + p_n(X)^3$.

Problem pRoblem prOblem proBlem probLem problEm probleM 3. For fOr foR A Set sEt seT S Of oF Positive pOsitive poSitive posItive posiTive positIve positiVe positivE Integers iNtegers inTegers intEgers inteGers integErs integeRs integerS And aNd anD A Positive pOsitive poSitive posItive posiTive positIve positiVe positivE Integer iNteger inTeger intEger inteGer integEr integeR n, Consider cOnsider coNsider conSider consIder consiDer considEr consideR The tHe thE Game gAme gaMe gamE Of oF (N, s)-*nim* (n, S)-*nim* (n, s)-*Nim* (n, s)-*nIm* (n, s)-*niM* Which wHich whIch whiCh whicH Is iS As aS Follows fOllows foLlows folLows follOws folloWs followS. A Pile pIle piLe pilE Starts sTarts stArts staRts starTs startS With wIth wiTh witH n Watermelons wAtermelons waTermelons watErmelons wateRmelons waterMelons watermElons watermeLons watermelOns watermeloNs watermelonS. Two tWo twO Players pLayers plAyers plaYers playErs playeRs playerS, Deric dEric deRic derIc deriC And aNd anD Erek eRek erEk ereK, Alternate aLternate alTernate altErnate alteRnate alterNate alternAte alternaTe alternatE Turns tUrns tuRns turNs turnS Eating eAting eaTing eatIng eatiNg eatinG Watermelons wAtermelons waTermelons watErmelons wateRmelons waterMelons watermElons watermeLons watermelOns watermeloNs watermelonS From fRom frOm froM The tHe thE Pile pIle piLe pilE, With

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wIth wiTh witH Deric dEric deRic derIc deriC Going gOing goIng goiNg goinG First fIrst fiRst firSt firsT. On oN Any aNy anY Turn tUrn tuRn turN, The tHe thE Number nUmber nuMber numBer numbEr numbeR Of oF Watermelons wAtermelons waTermelons watErmelons wateRmelons waterMelons watermElons watermeLons watermelOns watermeloNs watermelonS Eaten eAten eaTen eatEn eateN Must mUst muSt musT Be bE An aN Element eLement elEment eleMent elemEnt elemeNt elemenT Of oF S. The tHe thE Last lAst laSt lasT Player pLayer plAyer plaYer playEr playeR To tO Move mOve moVe movE Wins wIns wiNs winS. Let lEt leT $F(s)$ $f(S)$ Denote dEnote deNote denOte denoTe denotE The tHe thE Set sEt seT Of oF Positive pOsitive poSitive posItive posiTive positIve positiVe positivE Integers iNtegers inTegers intEgers inteGers integErs integeRs integerS N For fOr foR Which wHich whIch whiCh whicH Deric dEric deRic derIc deriC Has hAs haS A Winning wInning wiNning winNing winnIng winniNg winninG Strategy sTrategy stRategy strAtegy straTegy stratEgy strateGy strategY In iN (N, s)-nim (n, S)-nim (n, s)-Nim (n, s)-nIm (n, s)-niM. Let lEt leT T Be bE A Set sEt seT Of oF Positive pOsitive poSitive posItive posiTive positIve positiVe positivE Integers iNtegers inTegers intEgers inteGers integErs integeRs integerS. Must mUst muSt musT The tHe thE Sequence sEquence seQuence seqUence sequEnce sequeNce sequenCe sequencE T, $f(t)$, $f(f(t))$, $\ldots$ t, $F(t)$, $f(f(t))$, $\ldots$ t, $f(T)$, $f(f(t))$, $\ldots$ t, $f(t)$, $F(f(t))$, $\ldots$ t, $f(t)$, $f(F(t))$, $\ldots$ t, $f(t)$, $f(f(T))$, $\ldots$ t, $f(t)$, $f(f(t))$, $\circ..$ t, $f(t)$, $f(f(t))$, $.\circ.$ t, $f(t)$, $f(f(t))$, $..\circ$ Be bE Eventually eVentually evEntually eveNtually evenTually eventUally eventuAlly eventuaLly eventualLy eventuallY Constant cOnstant coNstant conStant consTant constAnt constaNt constanT?

Time tIme tiMe timE Limit lImit liMit limIt limiT: Four fOur foUr fouR Hours hOurs hoUrs houRs hourS Thirty tHirty thIrty thiRty thirTy thirtY Minutes mInutes miNutes minUtes minuTes minutEs minuteS.

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