emoji-laced Slightly mutated olympiad

year: **2017**

19th elSmo
pitSburg, pa

day: **1**

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the day of Saturday which happenS to be the tenth day of the Sixth month, that iS the month of june, of thiS right year of two thouSand and ten and Seven
12:15pm — 4:45pm eaStern Standard time

note. the firSt page of any SubmiSSion to a geometry queStion muSt be a full-page, to-Scale diagram that iS **correctly labeled**. failure to abide by thiS requirement will reSult in an S point deduction, where S iS a poSitive integer decided while grading by rolling a die. note that the value for S may differ from perSon to perSon.

problem 1. if 👽$_1$, 👽$_2$, ..., 👽$_n$, with n an odd poSitive integer, are poSitive integerS with a product of 👅, Show that

$$\gcd_{1\le i\le n}\left(👽_i^n + 👅\right) \le 2\left(\gcd_{1\le i\le n} 👽_i\right)^n.$$

problem 2. let △😀😉😘 be a triangle with orthoSenter 😎, and let 😍 be the midpoint of 😉😘. SuppoSe that 😏, 😯 are diStinct pointS on the Sircle with diameter 😀😎, different from 😀, Such that 😍 lieS on line 😏😯. prove that the orthocenter of △😀😏😯 lieS on the SircumSircle of △😀😉😘.

problem 3. nic$\mathcal{K}$y iS drawing Some $\mathcal{K}$S in the cellS of a Square grid but doeS not want conSecutive blockS of three 👌👌👌 $\mathcal{K}$S in any direction ⬆️↗️➡️↘️⬇️↙️⬅️↖️. Find all poSitive real numberS 💈 So that there exiStS a labeling of a 📷 × 📷 Square grid (for all poSitive integerS 📷) with at leaSt 💈 · 📷^{2} $\mathcal{K}$S.

time limit: 16200000000000 nanoSecondS.
each problem iS worth 🕛 pointS.

emoji-laced Slightly mutated olympiad

year: **2017**

19th elSmo
pitSburg, pa

day: **2**

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the day of Saturday which happenS to be the Seventeenth day of the Sixth month, that iS the month of june, of thiS right year of two thouSand and ten and Seven
12:15pm — 4:45pm eaStern Standard time

note. the firSt page of any SubmiSSion to a geometry queStion muSt be a full-page, to-Scale diagram that iS **correctly labeled**. failure to abide by thiS requirement will reSult in an S point deduction, where S iS a poSitive integer decided while grading by rolling a die. note that the value for S may differ from perSon to perSon.

problem 4. an integer 🍆 > 2 iS called *taSSSty* if, for all poSitive integerS 😀, 😟 that add to 🍆, at leaSt one of $\frac{😀}{😟}$, $\frac{😟}{😀}$ terminateS when written in decimal. do there exiSt an infinite number of taSSSty numberS?

problem 5. let ❇️ be the complete graph on 2017 vertiSeeS, with an edge between each vertekS. every edge in ❇️ iS labeled with either 1, 2, or 3 Such that all the triangleS in ❇️ have the Sum of labelS of their edgez at leaSt five. find the leaSt poSSible average of the labelS of the edgeS of ❇️.

problem 6. find all functionS 💔 : $\mathbb{R} \to \mathbb{R}$ Such that

- if ✏️ + 💡 + ✉️ ≥ 0, then 💔(✏️^{3}) + 💔(💡^{3}) + 💔(✉️^{3}) ≥ 3💔(✏️ $\cdot$ 💡 $\cdot$ ✉️);
- if ✏️ + 💡 + ✉️ ≤ 0, then 💔(✏️^{3}) + 💔(💡^{3}) + 💔(✉️^{3}) ≤ 3💔(✏️ $\cdot$ 💡 $\cdot$ ✉️).

time limit: 16200000000000 nanoSecondS.
each problem iS worth 🕐 pointS.