Evan Left MOp :(

28th ELMO
GREENSBORO, NC

Year: **2026**

Day: **1**

June 13, 2026
12:30PM-5:00PM EDT

Problem 1. Let P_1, P_2 and P_3 be distinct points on a circle Ω. For all positive integers $1 \leq i \leq 23$, define P_{i+3} as the reflection of P_i over line $P_{i+1}P_{i+2}$. Prove that $P_{20}P_{26}$ is tangent to Ω.

Problem 2. An $n \times n$ *Latin square* is an $n \times n$ grid of cells where each cell contains exactly one positive integer between 1 and n, each row contains the numbers 1 to n in some order, and each column contains the numbers 1 to n in some order. Call four numbers *stable* if there are either exactly two or exactly four distinct values among them. For which positive integers n is it possible to construct an $n \times n$ Latin square such that every quadruple of four distinct cells forming the corners of a grid rectangle has entries which are stable?

Problem 3. Determine all integers $n \geq 2$ such that there exists an integer polynomial $P(x) \in \mathbb{Z}[x]$ of degree at most $0.9n$ such that the range of P modulo n is the same as the set of zeros of P modulo n.

The range of P modulo n is the set of all integers $0 \leq y \leq n-1$ such that there exists a positive integer x where $P(x) - y$ is a multiple of n.

The set of zeros of P modulo n is the set of all integers $0 \leq x \leq n-1$ such that $P(x)$ is a multiple of n.

Time limit: 4 hours 30 minutes.
Each problem is worth 7 points.

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Year: **2026**

Day: **2**

June 18, 2026
12:00PM-4:30PM EDT

Problem 4. For an integer n, let $\tau(n)$ denote the number of positive integer divisors of n. For each positive integer k, let $S_k = \{\tau(k), \tau(2k), \tau(3k), \cdots\}$. If $S_a = S_b$ for two positive integers a and b, prove that $\tau(a^n) = \tau(b^n)$ for all positive integers $n \geq 1$.

Two sets are equal if and only if every number which appears in one set also appears in the other set.

Problem 5. Let $\mathbb{N}$ denote the set of all positive integers. Find all functions $f\colon \mathbb{N} \to \mathbb{N}$ such that

$$f\left(f(n)^m\right) = 2^{f(m)^n}$$

for all $m, n \in \mathbb{N}$.

Problem 6. Let n be a positive integer. There are $n(n+1)$ points chosen on the circumference of a circle, and each of these points is colored one of n colors such that each color appears exactly $n+1$ times. Elmo wishes to choose n non-overlapping arcs of the circle such that both endpoints of each arc are selected points of the same color, and this color is different for each of the n chosen arcs.

Prove that Elmo can select the arcs in at least $n+1$ distinct ways.

Time limit: 4 hours 30 minutes.
Each problem is worth 7 points.