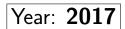




19th ELMO Pittsburgh, PA







Saturday, June 10, 2017 1:15PM — 5:45PM

Problem 1. Let a_1, a_2, \ldots, a_n be positive integers with product P, where n is an odd positive integer. Prove that

 $gcd(a_1^n + P, a_2^n + P, \dots, a_n^n + P) \le 2 gcd(a_1, \dots, a_n)^n.$

Problem 2. Let ABC be a triangle with orthocenter H, and let M be the midpoint of \overline{BC} . Suppose that P and Q are distinct points on the circle with diameter \overline{AH} , different from A, such that M lies on line PQ. Prove that the orthocenter of $\triangle APQ$ lies on the circumcircle of $\triangle ABC$.

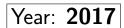
Problem 3. nic κ y is drawing kappas in the cells of a square grid. However, he does not want to draw kappas in three consecutive cells (horizontally, vertically, or diagonally). Find all real numbers d > 0 such that for every positive integer n, nic κ y can label at least dn^2 cells of an $n \times n$ square.





19th ELMO Pittsburgh, PA







Saturday, June 17, 2017 1:15PM — 5:45PM

Problem 4. An integer n > 2 is called *tasty* if for every ordered pair of positive integers (a, b) with a + b = n, at least one of $\frac{a}{b}$ and $\frac{b}{a}$ is a terminating decimal. Do there exist infinitely many tasty integers?

Problem 5. The edges of K_{2017} are each labelled with 1, 2, or 3 such that any triangle has sum of labels at least 5. Determine the minimum possible average of all $\binom{2017}{2}$ labels.

(Here K_{2017} is defined as the complete graph on 2017 vertices, with an edge between every pair of vertices.)

Problem 6. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that for all real numbers a, b, and c:

- (i) If $a + b + c \ge 0$ then $f(a^3) + f(b^3) + f(c^3) \ge 3f(abc)$.
- (ii) If $a + b + c \le 0$ then $f(a^3) + f(b^3) + f(c^3) \le 3f(abc)$.