vEry badLy naMed cOntest

Year: **2017**

19th ELMO
PITTSBURGH, PA

Day: **1**

Saturday, June 10, 2017
1:15PM — 5:45PM

Problem 1. Let $a_1, a_2, \ldots, a_n$ be positive integers with product P, where n is an odd positive integer. Prove that

$$\gcd(a_1^n + P, a_2^n + P, \ldots, a_n^n + P) \le 2\gcd(a_1, \ldots, a_n)^n.$$

Problem 2. Let ABC be a triangle with orthocenter H, and let M be the midpoint of $\overline{BC}$. Suppose that P and Q are distinct points on the circle with diameter $\overline{AH}$, different from A, such that M lies on line PQ. Prove that the orthocenter of $\triangle APQ$ lies on the circumcircle of $\triangle ABC$.

Problem 3. nicκy is drawing kappas in the cells of a square grid. However, he does not want to draw kappas in three consecutive cells (horizontally, vertically, or diagonally). Find all real numbers $d > 0$ such that for every positive integer n, nicκy can label at least dn^2 cells of an $n \times n$ square.

Time limit: 4 hours 30 minutes.
Each problem is worth 7 points.

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Year: 2017

19$^{\text{th}}$ ELMO
Pittsburgh, PA

Day: 2

Saturday, June 17, 2017
1:15PM — 5:45PM

Problem 4. An integer $n > 2$ is called *tasty* if for every ordered pair of positive integers (a, b) with $a + b = n$, at least one of $\frac{a}{b}$ and $\frac{b}{a}$ is a terminating decimal. Do there exist infinitely many tasty integers?

Problem 5. The edges of K_{2017} are each labelled with 1, 2, or 3 such that any triangle has sum of labels at least 5. Determine the minimum possible average of all $\binom{2017}{2}$ labels.

(Here K_{2017} is defined as the complete graph on 2017 vertices, with an edge between every pair of vertices.)

Problem 6. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that for all real numbers a, b, and c:

(i) If $a + b + c \geq 0$ then $f(a^3) + f(b^3) + f(c^3) \geq 3f(abc)$.

(ii) If $a + b + c \leq 0$ then $f(a^3) + f(b^3) + f(c^3) \leq 3f(abc)$.

Time limit: 4 hours 30 minutes.
Each problem is worth 7 points.