

Olympiad Training for Individual Study: Syllabus

OTIS Year IV: 2018-2019 academic year

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“It’s hard to do a really good job on anything you don’t think about in the shower.”
— Paul Graham

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§1 Overview

§1.1 Dates

Typically we follow MIT semesters¹.

Fall Early September to early-mid December.

Spring Mid-late January to mid-April (just before USAMO).

Problem sets and meetings are organized in two-week units on particular topics. Barring unexpected conflicts, snow days, apocalypse, etc. expect **six units each semester**.

§1.2 Formats (Weekly, Biweekly, Correspondence)

Starting in 2018, OTIS has a few different formats which differ only in the number of meetings.

Weekly The Weekly format consists of the following components.

- Every two weeks, a **unit** consisting of a couple worked examples, plus a **problem set** approximately 9-15 olympiad problems (of which you solve some subset), most of which focus in one particular topic. See Section 3.
- Prompt communication (via email, Facebook, Hangouts, etc.) for questions etc. See Section 3.4.
- A set of ten full-length **mock olympiads** (3.5 hours and 3 problems, USAMO/IMO style) and nine brief short-answer quizzes (for computation practice). See Section 4.
- Every week, an brief **online meeting** to discuss the current unit, usually over Google Hangouts. These meetings are short quite: **42 minutes long** (0.7 hours). See Section 5.

Units would be usually completed every two meetings. So the $(2k - 1)$ st and $2k$ th meeting would cover the k th unit ($k \geq 1$), and the corresponding problem set is due by the start of the $(2k + 1)$ st meeting. It is possible to move slightly faster or slower than this.

Biweekly The biweekly format is the same, but with meetings every two weeks, rather than every weeks. So the k th meeting would cover the k th unit ($k \geq 1$), and the corresponding problem set is due by the start of the $(k + 1)$ st meeting.

Correspondence The Correspondence format is the same, except that there are **no meetings** at all. Normally, you would submit problem sets before the start of meetings, but in the Correspondence format you would submit problem sets as you complete them (and would contact me as you did so). Consequently, the program would be more or less entirely self-paced, (and you can move through as many or as few units as you have time for).

This is recommended only if you are really motivated. But it has been done before, and there is no limitation on capacity.

¹You may drop OTIS mid-way at any point, though obviously I would appreciate advance notice.

§1.3 Prerequisites

- You should be comfortable qualifying for your country's national olympiad and be able to read and write proofs.
- You need to complete² the **math orientation packet** and submit it by the specified deadline. (This packet is sent to participants when their application is accepted.)
- It is a good investment³ to learn \LaTeX well in the summer before OTIS begins; it will pay back in spades when writing up solutions.
- Students can participate at most two full years in either the Weekly or Biweekly format.⁴ (Also, note that practice exams repeat every two years, so in a third year the practice exams would become redundant.)
- You should be prepared to spend a lot of time. A crude estimate for the total time commitment might be *8-12 hours per week*, though this varies substantially between different people and between units.

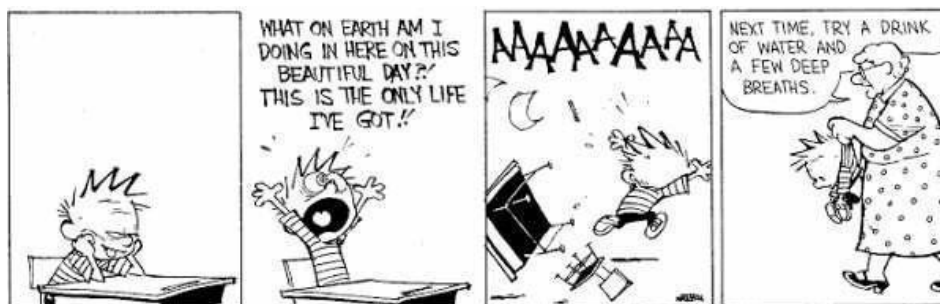
§1.4 TA meetings

Some of you might be surprised how little lecture time there is. For those of you who want more instructor time, I often contract one or more teaching assistants, who can meet with you separately (and who would be paid separately). See Section 5.4 for details.

§1.5 Payment information

The rate is $\$80(H + 4)$ each semester (where H is the number of hours of meetings).⁵ Assuming six units as advertised, each semester will have $H \in \{0, 4.2, 8.4\}$ depending on which format you are enrolled in.

Once enrolled, you'll get a "Parent Information" handout with details on payment methods and the like (the boring stuff).



²As part of this packet, you will need to work through the first three chapters of my textbook, <http://web.evanchen.cc/geombook.html>. This will cover fundamental olympiad geometry concepts such as angle chasing, radical axis, homothety.

³This suggestion is actually from one of my past participants, so I'm not just making this up.

⁴This is both for fairness, as well as limitations on my materials.

⁵The +4 term accounts for grading, answering questions, and preparing materials. And, uh, server costs, unbelievably enough.

§2 Curriculum and Materials

§2.1 Units

The page <http://web.evanchen.cc/static/otis-samples/synopsis.html> contains a catalog of possible units (which is always changing, so may be slightly out of date). The choice of which units to cover is done by mutual agreement (usually I make a recommendation but let you request any changes).

§2.2 Lecture notes (walkthroughs)

The lecture notes for every unit includes some “walkthroughs”: example problems together with a sequence of parts intended to take you step by step through the problems. Here is an example of what that might look like.

Problem (IMO 2003/6). Let p be a prime number. Prove that there exists a prime number q such that for every integer n , the number $n^p - p$ is not divisible by q .

Walkthrough. (a) Show that if $q \not\equiv 1 \pmod{p}$ then this fails. So we will restrict our attention to $q = pk + 1$.

(b) Prove that it's sufficient to have $p^k \not\equiv 1 \pmod{q}$, for the k in (a).

... and so on.

During meetings, I will probably pick one or two of the walkthroughs to work through with you. But even in Weekly format we more than likely won't get to finishing all the written walkthroughs, and in that case you can (and should) read through the rest of the walkthroughs on your own time. Full solutions to all the walkthroughs are provided in an appendix at the end of each unit.

§2.3 OTIS-WEB

Starting in the 2017-2018 school year, OTIS has moved onto its own dedicated website⁶. The URL for creating an account will be emailed to you upon completion of the summer packet.

The website is sort of the central hub for all materials It will be used to manage the following:

- Materials (notes and problem set) for each unit,
- Transcripts of meetings, and
- Your submissions to the problem sets.

Mock olympiads and quizzes are submitted separately, see below. (They are still listed on OTIS-WEB.)

Note that lecture notes for meetings are uploaded well in advance, and you are encouraged to skim through these beforehand. In particular, for Traditional students, certain lectures may have associated **reading you will need to do before-hand**; these will be clearly indicated in the lecture notes PDF. (You are also welcome/encouraged to work through some of the example problems yourself before the lecture.)

All materials are *internal use only*.

⁶Before, I used an ad-hoc Google Drive system.

§3 Problem sets

The most important part of the training is the regular problem sets.

§3.1 Solving instructions

Each problem set has 10-15 problems, but you're not expected to solve all of them: each problem has a weight (like [5♣]) attached to it, and you are aiming to solve a certain target score or more. This will be self-explanatory once you see your first problem set.

You are expected to communicate with me throughout the week about any problems you have difficulty with (this should happen every week). A good rule of thumb is to **ask for hints after an hour with no progress**. When asking for hints, please describe the progress you've made so far, or approaches that you've tried and didn't work. See Section 3.4 for more on that.

§3.2 Write-ups and submission

You only need to submit outlines of solutions (although full write-ups are certainly welcome if you have the time). This way I can see which problems you have solved and roughly by what method, and also sanity-check that there are no obvious errors in the solution. An "outline" is just 2-5 sentences that capture the main idea.

Here is an example of what I mean:

Problem (HMMT 2016 Guts 17). Compute the sum of all integers $1 \leq a \leq 10$ with the following property: there exist integers p and q such that p , q , $p^2 + a$ and $q^2 + a$ are all distinct prime numbers.

Solution. Odd a fail for parity reasons and $a \equiv 2 \pmod{3}$ fail for mod 3 reasons. This leaves $a \in \{4, 6, 10\}$, for which we can take $(p, q) = (3, 5), (5, 11), (3, 7)$, respectively. Thus $4 + 6 + 10 = 20$. \square

As I mentioned in the beginning, I suggest typesetting your problem sets in \LaTeX . The output is very pretty, you learn how to use \LaTeX (useful later), and you now have digital copies of all your work. After all, given how much time you're spending on my problems, don't you want to keep records of them?

In any case, for each unit's problem set, you should write up solutions and submit problem sets by uploading a single PDF to OTIS-WEB. You should upload it under the "**File Uploads**" page for that unit.

§3.3 Due dates

To reiterate Section 1.2, problem sets are due differently depending on which format of OTIS you are taking.

- For the Weekly and Biweekly formats, problem sets are due each two weeks. In particular:
 - In the Weekly case, the $(2k - 1)$ st and $2k$ th meeting would cover the k th unit ($k \geq 1$), so the corresponding problem set is due by the start of the $(2k + 1)$ st meeting.
 - In the Biweekly case, the k th meeting would cover the k th unit ($k \geq 1$), and the corresponding problem set is due by the start of the $(k + 1)$ st meeting.

- In the Correspondence case, you are self-paced: there is no hard due date. Each time you submit a problem set, you should then contact me to review your submissions; I will then advance your counter (thus revealing the solutions and unlocking another unit).

For transparency, I should mention that the regular problem set submissions are *not* graded line-by-line⁷; sorry, there are just too many. Usually, I read through the solutions to sanity-check them and point any obvious issues I can see (wrong final, common pitfalls, etc.). So I will probably not notice typos or small arithmetic errors, for example.

(The practice exams are graded fully, though.)

§3.4 (Important) Talk to me, I don't bite

I want to really stress the importance of keeping in contact with me, and in particular the importance of asking for help. The problem sets are meant to help you learn through practice, rather than judgment (of skill or time spent or whatever). A problem will usually teach you a lot more in the first one or two hours than in the six hours after that. So it's much better to ask for directions once you've hit a barrier, rather than being stuck for hours on problem 5 and then not having enough time to try problems 6-10.

On a typical unit, a rough estimate is that you will need help on at least 1/3 of the problems, in the sense that you're unlikely to solve the problem no matter how much time you spend on it. (The actual constant varies a lot by person and unit.) That means that in expectation you should be writing to me 3+ times per unit. This is a lot!⁸

Put more concisely, **the problem sets should feel interactive**. Of course this only works if you reach out to me.

Some pointers on asking for help:

- Do it. Really. I want to hear from you.
- You can contact me through any of Facebook, email, or Google Hangouts. I recommend one of the first two since I'll see those on my phone⁹, but I'll eventually notice and respond to Google Hangouts messages as well.
- *Timing*: I make an effort to try to respond to OTIS messages as soon as I see them, since I consider this to be one of the most important parts of my job. However I tend to sleep early (sometimes as early as 10PM ET) and so if you send me a message late at night I will probably not get it until the next morning. But if you send me a message during daylight hours, you might typically expect a response within about 6 hours.

If I don't respond to something within 24 hours, that is not okay on my part; please remind me.

- It's more convenient for me if you refer to a problem by source when given, e.g. "USAMO 2010/6" (instead of "problem 10", say).
- Describing what you've tried so far (even things that didn't work) is helpful. That will help me give you more refined suggestions.

⁷This is part of why I only ask for outlines of solutions.

⁸If you find you're consistently able to solve nearly all the problems, consider asking me to make your curriculum more difficult.

⁹Hooray for technology in education?

- Don't be afraid to ask for more directions if the first hint I give isn't sufficient! (I tend to give conservative hints, to avoid spoiling too much.)

That +4 term in the semester payment is there for a reason. Make use of it!

§4 Mock olympiads and quizzes

Both the mock olympiads and the short quizzes will be **graded through the Grade-Scope program**. (You'll be set up for GradeScope when submitting the math orientation packet.) The exams (and solutions) themselves will be shared on OTIS-WEB. The solutions are actually available immediately, so you can read them right after taking the test, without waiting for grading.

You can disregard the “code-names” for the olympiads (one of “Foxtrot”, “Tango”, “Waltz”). These are for my internal use.

§4.1 Strategy

Once you are registered for OTIS, you will find on the portal a handout about meta-strategies for test taking on olympiads (with respect to write-ups, problem ordering, etc). I strongly recommend reading through this before taking your first mock olympiads, since this will give you a chance to practice these strategies on many practice tests before the first one.

§4.2 Dates

There will be 10 tests (mock olympiads) in the school year, which come in pairs. The short quizzes are short-answer practice, each with 6 much quicker short-answer problems. The dates for the practice exams are specified in Table 1.

| Test Name | Released | Due |
|--------------------------------------|----------|-------|
| Tests 1 and 2 | Aug 5 | Oct 1 |
| Tests 3 and 4 | Sep 5 | Nov 1 |
| Quiz A | Oct 2 | Nov 1 |
| Tests 5 and 6 | Oct 5 | Dec 1 |
| Quiz B | Nov 2 | Dec 1 |
| Quiz C | Nov 2 | Dec 1 |
| <i>Fall bonus final (optional)</i> | Dec 14 | |
| Quiz D | Jan 2 | Feb 1 |
| Quiz E | Jan 2 | Feb 1 |
| Quiz F | Jan 2 | Feb 1 |
| Tests 7 and 8 | Jan 5 | Mar 1 |
| Quiz G | Feb 2 | Mar 1 |
| Quiz H | Feb 2 | Mar 1 |
| Tests 9 and 10 | Feb 5 | Apr 1 |
| <i>Spring bonus final (optional)</i> | Mar 14 | |

Table 1: Schedule for OTIS practice exams

I run grading at the beginning of each calendar month, but actually for all the practice tests that are still open — meaning there are two grading sessions for every month. For

example, Test 3 is released on September 5, and due on November 1. So if you submit close to the deadline, you would get feedback by the first week of November. However, if you submit early by October 1, you would actually get the feedback by the first of October, before the test is due! (This would be around the same time most of the Tests 1 and 2 are graded.)

§4.3 Instructions for tests (mock olympiads)

Here are some instructions for submissions.

- Each test is three problems, 7 points per problem.
- There are **three levels: JMO, USAMO, and TST**. You should pick one of them to take.
- The time limit is **only 3.5 hours**. (This is deliberately shorter than the 4.5 hours used for JMO / USAMO; to help you get used to working under time pressure, and to make it easier to fit in your schedule.)
- Like a real olympiad, submit solutions on the given answer sheets. You can download the answer sheets from OTIS-WEB.
- You are encouraged to do the problems on the olympiad that you didn't take, but please do not submit those solutions on GradeScope; leave them blank.¹⁰
- When submitting mock olympiads, be careful to submit the right papers to the right problems. There are five distinct questions among the tests, of which you will only submit three.

§4.4 Instructions for quizzes (short answer)

These are intended to give you bit of short-answer practice during the school year, to help prepare you for AIME, HMMT, PUMaC, MP4G, et cetera. (Indeed, you can expect most of the problems to come from there.)

- Each quiz is **45 minutes long**, and features **6 short answer problems**. Think of it like $\frac{2}{5}$ of an AIME, except probably harder and faster.
- Use the answer sheet provided on OTIS-WEB. You will only submit short answers for grading. (Thus your score will be one of $\{0, 1, \dots, 6\}$.)

Most of the problems will be from past NIMO and OMO contests.

§4.5 Bonus final olympiads

You can request a final at the date in Table 1 if you have completed most of the previous OTIS exams due before then. (I'll send out an email to everyone eligible at that time.) So, it serves as sort of an end-of-semester present, which is why it comes out around Christmas in the fall semester! The final problems are supposed to especially nice (or at least I think so, seeing as I save them all year). I hand-design these for each individual.

The bonus finals are totally optional. You can do these as a mock exam if you really want to, or you can also just work on the problems for fun. (But if you do want to take it under exam conditions, then email me your solutions and I will grade them.)

¹⁰It seems that the GradeScope now lets you omit submissions for problems; this was not true before.

§5 Meetings

§5.1 Logistics

We will use [Google Hangouts](#) for meetings. My Gmail address is

chen.evan6@gmail.com

Please be online on Hangouts at the scheduled meeting time.

§5.2 Transcripts

Typically, I will have a \LaTeX document (in real time¹¹) which serves as a sort of “white board” for the class. So problem statements, partial progress, etc. will all appear there.

Here is an example of part of a transcript from last year, also posted in full on my website.¹²

IMO 2014 Problem 6

Greedy Algorithms
Evan Chen

Example (IMO 2014/6)

Prove that for all sufficiently large n , in any set of n lines in general position it is possible to colour at least \sqrt{n} lines blue in such a way that none of its finite regions has a completely blue boundary.

Strategy

Color lines blue until stuck.

Proof this strategy works.

Look at a maximal configuration. Claim that in here, at least \sqrt{n} lines are blue.

So suppose there k blue lines and $n - k$ red lines. Then there are $\binom{k}{2}$ intersections of two blue lines. Moreover every red line is part of an almost-blue polygon. So can associate every red line to a blue intersection.

By “geometry”, at most two red lines per blue vertex. Thus

$$\binom{k}{2} \geq \frac{1}{2}(n - k) \implies k \geq \sqrt{n}.$$

Putnam example

/home/evan/Documents/Teaching/Tutoring2016/zack-local/local-args-script.pdf [4/8]

The advantage of this setup is that we get a full transcript of the class that you can review on your own time. I usually upload these immediately after your class.

You can review the transcripts at any time later; the transcripts will be under the link “File Uploads” for each unit.

§5.3 What happens in a meeting?

Participants meet every one or two weeks. The exact time of day is usually coordinated close to the start of each semester.¹³

Examples of things we can do in a meeting on the n th unit (all flexible):

¹¹This means a lot of me typing furiously and getting angry at \LaTeX compilation errors. I always forget the `fragile` option when using Asymptote in Beamer.

¹²Yes, I really type \LaTeX that fast.

¹³Fair warning: I have a lot of participants, so I re-schedule classes relatively often. I apologize in advance.

- Present solutions to problems from the previous $(n - 1)$ st unit.
- Work together through the examples/walkthroughs for the n th unit.
- Work together through any practice problems from the n th unit (for example, things you've tried and are stuck on).
- Questions about reading or lecture notes (or anything else).

§5.4 Meetings with teaching assistants (optional)

Since the meetings with me are so short (or non-existent in the Correspondence format), I am often able to pair you with an assistant¹⁴. This is completely voluntary, but the idea is that you could have more instructional time with me: for example, if you met with me every other week, you could also have a one-hour meeting with the assistant during the alternating weeks. (I do this because past participants have indicated this is helpful for finishing the problem sets, and more enjoyable.)

Often in these meetings, the TA's would help work with you on practice problems. But you can with them about anything else too. Scheduling and payment is coordinated directly with the TA¹⁵ and in particular the length of a TA meeting can be whatever you agree on.

§6 Surveys/feedback

Throughout the year, I send three OTIS surveys by email.

- The first (longest) survey¹⁶ is sent around the third unit or so.
- The second survey will be sent around the end of the fall semester.
- The third (shortest) survey will be sent at the end of the year.

You need not worry about these until you see them, but when you do, I really appreciate your feedback.

Of course if you have comments at other times of the year, I am more than happy to take those as well.

In particular, if you run into a problem you think would fit well on one of my units, please send it to me! (Or if you run into a nice problem in general.) Many of the problems you see in OTIS were forwarded to me by students and friends.

¹⁴In terms of qualifications: typically I contract OTIS alumni who have won USAMO or made IMO.

¹⁵The rate is often the same. I don't take a cut, other than asking the TA's to help me with grading.

¹⁶The nice thing about having a start-of-year survey (rather than an end-of-year survey like the rest of the world) is that your feedback will actually be used right away this year, rather than just helping future participants.

§7 Your to-do checklist

Here is a checklist of what's expected of you, to help keep track of the many parts.

- After acceptance:
 - Complete the summer math orientation packet
- After submitting the summer packet:
 - Register for an account on OTIS-WEB
 - Agree on a schedule for the school year
 - Read the handout on test-taking strategy (before first practice olympiad)
- Before each unit:
 - Do any reading specified in the notes for that unit
 - Optionally, spend a little time on the example problems beforehand
- During each unit:
 - Meet with me at your scheduled time (Weekly or Biweekly format)
 - Meet with TA (if you opted to be paired with one)
 - Work through the walkthroughs
 - Solve the problem set for that unit
 - Keep me updated on your progress on the problems (see Section 3.4)
 - Submit outlines of solutions to OTIS-WEB (under “file uploads”)
- Before the deadlines in Table 1, submit:
 - Practice olympiads (usually two per month)
 - Quizzes (in selected months)
 - Request a bonus final (optional)
- Throughout the year:
 - Respond to surveys
 - Send me good problems