Remarks on English

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Exposition, criticism, appreciation, is work for second-rate minds.

— G. H. Hardy

§1 Grading

Your score on an olympiad problem is a nonnegative integer at most 7. The unspoken rubric reads something like the following:

	Description
7*	Problem was solved
6	Tiny slip (and contestant could repair)
5	Small gap or mistake, but non-central
2	Lots of genuine progress
1*	Significant non-trivial progress
0*	"Busy work", special cases, lots of writing

The "default" scores are starred above. Note that, unlike high school English class or the SAT essay, you don't get points just because you wrote a lot!

In theory, your solutions to olympiads are graded solely based on math. In practice, style still does play a role in some ways: the harder your solution is to understand, the less likely the grader is to understand you, and the less likely you are to earn points you deserve.

In addition, poorly written solutions make the graders sad, and you wouldn't want that, would you?

§2 Stylistic suggestions

Here are some tips of mine that I don't think are stressed enough.

§2.1 Never write wrong math

This is much more of a math issue than a style issue: you can lose all of your points for making false claims. Personally, I often *stop reading* a solution if it makes an egregiously false claim: if someone claims that some fixed point is the incenter of ABC, when it's actually the arc midpoint, then I know the solution isn't going to have any substantial progress.

As a special case, don't say something that is partially true and they say how to fix it later. At best this will annoy the grader; at worst they may get confused and think the solution is wrong.

§2.2 Emphasize the point where you cross the ocean

Solutions to olympiad problems often involve a few key ideas, with the rest of the solution being checking details. You want graders to immediately see all the key ideas in the solution: this way, they quickly have a high-level understanding of your approach.

Let me share a quote from Scott Aaronson:

> Suppose your friend in Boston blindfolded you, drove you around for twenty minutes, then took the blindfold off and claimed you were now in Beijing. Yes, you do see Chinese signs and pagoda roofs, and no, you can't immediately disprove him — but based on your knowledge of both cars and geography, isn't it more likely you're just in Chinatown? ... **We start in Boston, we end up in Beijing, and at no point is anything resembling an ocean ever crossed.**

Olympiad solutions work the same way: a geometry solution might require a student to do some angle chasing, use Fact 5 to deduce that two triangles are congruent, and then finish by doing a little more angle chasing. In that case, you want to highlight the key step of proving the two triangles were congruent, so the grader sees it immediately and can say "okay, this student is using this approach".

Ways that you can highlight this are:

- Isolating crucial steps and claims as their own **lemmas.**[1]
- Using **claims** to say what you're doing. Rather than doing angle chasing and writing "blah blah blah, therefore $\triangle M_BI_BM \sim \triangle M_CI_CM$", consider instead "We claim $\triangle M_BI_BM \sim \triangle M_CI_CM$, proof".
- **Displaying** important equations. For example, notice how the line

  $$\triangle M_BI_BM \sim \triangle M_CI_CM \tag{1}$$

  jumps out at the reader. You can even number such claims to reference them letter, e.g. "by (1)". This is especially useful in functional equations.
- Just **say it**! Little hints like "the crucial claim is X" or "the main idea is Y" are immensely helpful. Don't make X and Y look like another intermediate step.

§2.3 "Find all..."

Many problems will ask you to "find all objects satisfying some condition" (for example, functional equations, Diophantine equations). For any solution of this form, I strongly recommend that you structure your solution as follows:

- Start by writing "**We claim the answer is ...**".
- Then, say "**We prove these satisfy the conditions**", and do so. For example, in a functional equation with answer $f(x) = x^2$, you should plug this f back in and verify the equation is satisfied. Even if this verification is trivial, you must still explicitly include it, because it is part of the problem.
- Finally, say "**Now we prove these are the only ones**" and do so.

[1] This is often useful for another reason: breaking the proof into individual steps. The complexity of understanding a proof grows super-linearly in its length; therefore breaking it into smaller chunks is often a good thing.

Similarly, some problems will ask you to "find the minimum/maximum value of X". In such situations, I strongly recommend you write your solution as follows:

- Start by writing "**We claim the minimum/maximum is . . .** ".
- Then, say "**We prove that this is attainable**", and give the construction (or otherwise prove existence). Even if this verification is trivial, you must still explicitly include it, because it is part of the problem.
- Finally, say "**We prove this is a lower/upper bound**", and do so.

Failing to do one of the steps mentioned above is a classic newbie mistake. Make it abundantly clear to the grader that you know the difference between a bound and a maximum.

§2.4 Leave space

Most people don't leave enough space. This makes solutions hard to read.

Examples of things you can do:

- Skip a line after paragraphs. Use paragraph breaks more often than you already do.
- If you isolate a specific **lemma or claim** in your proof, then it should be on its own line, with some whitespace before and after it.
- Any time you do **casework**, you should always split cases into separate paragraphs or bullet points. Make it visually clear when each case begins and ends.
- Display important equations, rather than squeezing them into paragraphs. If you have a long calculation, then do an aligned display[2] rather than squeezing it into a paragraph. For example, instead of writing $0 \le (a-b)^2 = (a+b)^2 - 4ab = (10-c)^2 - 4\left(25 - c(a+b)\right) = (10-c)^2 - 4\left(25 - c(10-c)\right) = c(20-3c)$, write instead

$$\begin{aligned} 0 &\le (a-b)^2 \\ &= (a+b)^2 - 4ab \\ &= (10-c)^2 - 4\left(25 - c(a+b)\right) \\ &= (10-c)^2 - 4\left(25 - c(10-c)\right) \\ &= c(20-3c). \end{aligned}$$

§2.5 Other things

Try to have nice handwriting. Include a large, scaled diagram in geometry problems[3]. Leave 1-inch (or more) margins. Write your proofs forwards even if you solved the problem backwards. If you need to cite a theorem, say clearly how you're doing so. Use variable names at your discretion. Strike out and cross out unwanted parts of your solution (don't scribble).

I'm sure someone has told you these before. If not, consider reading https://www.artofproblemsolving.com/articles/how-to-write-solution.

[2] This is the `align*` environment, for those of you that like LaTeX.

[3] And try to not have circles which look like potatoes.

§3 Example

Consider the following problem.

> **(USAMO 2014)** Let a, b, c, d be real numbers such that $b - d \geq 5$ and all zeros x_1, x_2, x_3, and x_4 of the polynomial $P(x) = x^4 + ax^3 + bx^2 + cx + d$ are real. Find the smallest value the product
>
> $$(x_1^2 + 1)(x_2^2 + 1)(x_3^2 + 1)(x_4^2 + 1)$$
>
> can take.

Here are two ways you could write the solution.[4]

> **Pretty poor solution.** $x_j^2 + 1 = (x - i)(x + i) \forall j \implies \prod x_j^2 + 1 = \prod (x_j + i)(x_j - i) = P(i)P(-i)$ so $(b - d - 1)^2 + (a - c)^2$. $\because x_j = 1 \to 16$ and $\binom{4}{2} - 1 = 5$. $b - d \geq 5$, so ≥ 16.

> **Better solution.** The answer is $\boxed{16}$. This can be achieved by taking $x_1 = x_2 = x_3 = x_4 = 1$, whence the product is $2^4 = 16$, and $b - d = \binom{4}{2} - 1 = 5$.
>
> Now, we prove this is a lower bound.
>
> The key observation is that
>
> $$\prod_{j=1}^{4} \left(x_j^2 + 1\right) = \prod_{j=1}^{4} (x_j - i)(x_j + i) = P(i)P(-i) = |P(i)|^2.$$
>
> Consequently, we have
>
> $$\begin{aligned} \left(x_1^2 + 1\right)\left(x_2^2 + 1\right)\left(x_3^2 + 1\right)\left(x_1^2 + 1\right) &= (b - d - 1)^2 + (a - c)^2 \\ &\geq (5 - 1)^2 + 0^2 = 16. \end{aligned}$$
>
> This proves the lower bound.

These solutions have the same mathematical content. But notice how in the better solution:

- The second solution makes it clear from the beginning what the answer is, and what the equality case is. (The first solution mixes these together.)
- Moreover, the main idea (of factoring with i) is explicitly labeled, so that even if you have never seen the problem before, you can tell at a glance what the main idea of the solution is.
- The equations are displayed in the second solution, making them much easier to read than in the first.

The second solution, despite being twice as "long", is by far faster to read than the first solution. In this case, the difference is not so bad because the problem and solution are quite short. However, in more involved problems the "not-so-good solution" becomes the "completely unreadable solution".

[4] Former solution worsened June 2018, with suggestions from Mitchell Lee.