Spring 2014 Executive Report
or: How to Write an OMO

Evan Chen

November 2013 - April 2014

This is some documentation about the process of preparing the 2014 Spring Online Math Open. It is intended to help future contest directors in preparing for future contests. We hope it also helps others aspiring to organize their own contests with an idea of how to proceed.

Contents

1. General Remarks

1.1. Mission Statement

Every leader of the Online Math Open will likely have a different vision for the purpose of the Online Math Open, and as such what follows is not an official mission statement (none exists), but rather my interpretation of it.

Above all, the Online Math Open is about sharing nice problems with the rest of the contest community. I am not interested in identifying the top scoring teams, I'm interested in the problems.[1] In other words, the over-arching goal above everything is problem quality.

I believe that the Online Math Open promotes problem-solving and problem-writing as something done by people. The problems are all visibly identified by author, and this is intentional – I want the contestants to see that the problems are written by other high schoolers from all over the United States. In that sense, it is a way of giving back.

1.2. Development Goals

Here are the goals which I set forth in the inaugural email for the Spring 2014 contest.

1. I want to have *at least 60 problems written* no later than *February 3, 2014*. This gives us the flexibility to assemble a contest that is balanced in both topics and difficulty.

2. We should prepare a *penultimate draft* of the problems within four weeks, by *March 3, 2014*. Over these four weeks we will be selecting and editing the problems to appear on the contest. By the deadline, we should commit to a selection of thirty problems.

3. Over the next four weeks, I want the contest to be extensively proofread. In particular, I would like every problem on the penultimate draft to be *read and test-solved by at least two people other than the author*. By the end of this process, there should be *no wrong answers* and *as few places for ambiguity as possible*.

4. The *final draft of the problems* should be finished no later than *March 31, 2014*.

5. The *final draft of the solutions* should be finished within *two days of the end of the contest*. That means solutions should primarily be written between the completion of the penultimate draft and the start of the contest. This is about four weeks to write 30 solutions. We should then rapidly add any new solutions from the forum and publish quickly.

6. *Publicity* should be started no later than *March 14, 2014*. I hope that this contest we can get at least *250 teams*.

[1] In particular, you might notice problems placed on the test which can be bashed or solved by engineer's induction or some equally ugly means. This is IMO not a huge loss. A nice problem will make the test, period. I just might try and edit it.

I hope these goals are reasonable. I estimate that the hardest part will be the problem-writing. It is actually IMO fairly difficult to come up with one high-quality problem a week, and there are only nine weeks. At the moment, there are only seven left-over problems from last year.

This was designed to combat some issues from last year, including the following.

- Last year, I was editing problems up to four hours before the contest started (where I said "convex quadrilateral" by habit on a nonconvex quadrilateral). This kind of timing is not acceptable. This year we are starting 18 weeks ahead of time instead of 10 weeks, so we have no excuse.
- New problems were being written within a few days of the contest to fill in gaps in difficulty and subject. In short, we had a problem shortage. This year, by having a surplus of problems, this should be a non-issue. Because nine weeks have been given for problem writing this time, and because we have winter break and more writers, we have no excuse.
- Solutions did not come out until three weeks after the contest because I was lazy and used the excuse of waiting for forum solutions. I want the solutions to be basically done by end-day this time around. Because we have plenty of time to write solutions, and because I actually committed to this ahead of time, we have no excuse.
- Publicity was not started until one or two weeks before the contest due to the website not being ready. This year, we have no excuse.

1.3. Collaborative Infrastructure

The OMO had a lot of infrastructure already set up from previous years; new contests will need to think this part out.

The two tools we used most were **Dropbox** (for sharing PDF's, etc.), and **KARL**. We actually get by without using Google Docs much; the main use of Google Docs was spreadsheets for answer checking.

KARL is the problem-writing server which I wrote for the NIMO and OMO. It's kind of hacked together, but works well enough. I wrote it to replace the dreadful unhighlighted page-long Google docs that had been used for Winter 2013; it was hard even to read a problem without reading its solution, let alone move problems around en-masse or look at topics and difficulties easily. I designed KARL to solve all these problems. So far it seems to be working great, as long as the web host it is placed on is stable.[2]

Some features of KARL include:

- Problems can be assigned subjects (a short string like "A") and difficulties (a positive integer). Problems can be sorted by difficulty.[3]
- Comments system. It is very primitive, just a text box where people can write things. It is customary to leave a brief "–Evan" or something related since the comments really are that primitive.

[2] This was not always the case. I had originally hosted KARL on a free web server, which turned out to have intolerable downtimes.

[3] This is something I really, really wish HMMT had.

- Voting system, where users can $+1$ or -1 a problem; the vote-count, if nonzero, is displayed next to the problem. It is customary to make a note in the comments when you $+1$ and -1 since the system does not record this.
- Multiple sets where problems can be moved around. For the OMO, we have a set called "OMO Storage", then a set for every contest.
- LaTeX support via MathJax.
- Solution and answer boxes which are initially "hidden" to prevent spoilers.
- User logins. It is possible to set up the system so that only certain users can view certain sets, so multiple contests can run on one KARL instance.

Dropbox handles the PDF's of the test, etc., as well as solution-writing later on. That is basically it. All the problem-writing happens on KARL.

Beyond that, we always have a fairly large e-mail thread. This is important for prodding and other communication, because no one will check KARL more than they check e-mail.

1.4. The Dictatorship

Politically, the Online Math Open follows the model of a **benevolent dictatorship**. That is, one person (the dictator) takes charge of conceptualizing the entire process, and then relays instructions to the team. In general, the dictator does not consult other members of the team before making a decision.

Of course, the dictator should be open to suggestions, but in my experience this is generally something that OMO dictators are good about as the team members usually know each other well. Larger organizations may need to be more cautious.

The primary advantage of the dictatorship is that it is **highly efficient and decisive**. It is efficient because decisions can be made instantaneously by someone with complete knowledge of the entire contest. It is decisive in the sense that no one is unsure what they should be doing; the dictator tells them what to do.

The primary drawback of this method is that it places significant strain on the dictator. The dictator can and should assume that **nothing is done unless explicitly requested and confirmed as completed**. This means e.g. the dictator should assume that problems are not being written unless he/she actually sees the new problems appearing. As a result, the dictator must assume complete responsibility for every aspect of the contest development, ranging from problem-writing to test-solving to publicity to things as simple as uploading the actual test[4].

Your role as dictator is to set a good example; no one will work if the dictator isn't working.

[4] I averted a small crisis in the NIMO when I realized, six hours before the April Round was to start, that no one had actually uploaded the PDF to the website yet

2. Summary of Events

2.1. Initialization

Development for the Spring 2014 OMO was kicked off on November 27, 2013 when I sent out a big email to everyone and invited two additional problem writers, Robin Park and Sammy Luo. Due to a wrong email address, Robin did not actually receive the invitation until very late in the development.

2.2. Problem-Writing Phase

As is usually the case with very early emails, few problems were actually written during the upcoming December. In retrospect, I think this is largely my fault – I did not have much in terms of problem ideas at the time, and so by not actually acting on my own words I lost the momentum that the kick-off announcement should have brought. This is a good example of how hypocrisy is especially bad in these types of organizations.

At the start of January I started prodding again, and since by this time I had a bunch of proposals to submit the prod was a lot more effective. I got some discussion of new problem ideas to happen in the email thread, which lead to the creation of some of the nice hard problems, like #20, #21, #22, and #29. I wonder if I should set up some other structure where people can just discuss problem ideas. KARL was designed for completed problems. Maybe a private AoPS forum would do the trick here. . .

In the spirit of procrastination, February was a mad rush to fill in problem gaps, now that we were behind the deadline. It was fortunate that problem quality didn't seem to noticeably dip but I would not count on this happening again. We did have a nice number of problems but the difficulty was concentrated towards the hard end, meaning we now had a shortage of easy problems. This seems to be a recurrent issue. I am forced to make up some things quickly – "filler" problems – to try and close up the gap. I personally hate filler problems and think they should not exist, but you have to do what you have to do. Though it's interesting that note that in the process of trying to invent some non-terrible fillers, I did get some good things out. People work incredibly well with deadlines.

Things looked okay by the end of February. I wrote

> We're in fairly good shape right now in terms of problems since we have a 20% margin now. If we could get one or two more problems from everyone (more or less of any difficulty/subject) then we'll be very well off (probably a 50% margin at that point, which would be fantastic). I'm hoping to hold onto the self-imposed goal of having a problem draft selected by March 3, 2014, so I'll probably start moving problems around over the weekend.

We did not in fact get the "one or two more problems" but nonetheless we had a reasonable margin to select a decent contest.

2.3. Publicity

Publicity was a disaster. First of all I completely forgot about this until Michael pointed out on March 14 that I had set a deadline of March 14 for publicity. In response I wrote

> Oh shoot I forgot about this, thanks for holding me to my word. I don't think we're doing much differently from last time but I might not have time over the next few days to actually carry this out.
>
> So the things that need to get done are
>
> - Posts on AoPS in AMC, OMO, WOOT forums. I don't have privileges in the WOOT forum but I can do the one in the AMC/OMO forums.
> - Facebook status.
> - Emails to people.
>
> I'd appreciate it if someone else could take charge of the emails as probably Ray/Victor know better than I do how coordination for that works. I can have the NIMO send an email out to everyone registered on io.org; this should hit a lot of people.
>
> I have to leave pretty soon but I made a doc for the publicity so that we can decide what we're going to say before posting. I think we might get more attention if we don't straight c/p from last year but hmm idk.

The clause I wrote about emails is a prime example of what *not* to do as dictator, since the emails were never written. The Facebook and AoPS forums, however, did go through.

2.4. Test-Solving

Now, the set has been finalized. At this point I quickly request that the OMO website code is edited so that the answers appear 24 hours after the deadline, then proceed to coordinate the test-solving.

As usual, we set up a spreadsheet for coordinating the test-solving. Unfortunately, no one likes test-solving, so it is somewhat of a battle to get people to check problems. Similarly, it is quite difficult to convince people to write solutions to problems; this is not such a huge deal since I am very fast with Vim and so doing the majority of solution-writing does not take too much time. I feel like gloating about this, so in Appendix A I have included a copy of the spreadsheet.

Fortunately, we get this done, but not without a broken problem (#17).

2.5. Contest Day

Or "days" if you are pedantic. This is usually a less stressful time. Just watch the clarifications roll in and enjoy the scoreboard.

I say "usually" because this time two things went very wrong.

- We actually had a broken problem – the "convex" pentagon in problem 17 did not actually exist. The problem originally asked for a convex $AXYZB$, but in fact, we need $AXYBZ$.
- A bug in the site caused the answers to be displayed twenty-four hours *early* rather than twenty-four hours late.

I think the first one could have been reasonably avoided with better testing. The second issue is something that seems like it could not have been reasonably avoided. (It's easy to say "check the code" or "test the website" but that would be hindsight bias.)

By Murphy's Law, most teams log in towards the end of the contest. Fortunately, the issue was reported soon enough that I was able to rush in and change all the answers stored in the site to -1. I then looked at the logs of teams which had logged in. The answers were not exactly conspicuously displayed, so most teams which did log in had not noticed them, only noticing that there were now "correct/incorrect" gradings on the site. The one team that did notice, fortunately, already had all the correct answers prior to the bug. Using the server logs I was (barely) able to prove that no other team could have placed in the top eighteen on the basis of the leaked answers. I do not know what I would have done if I had been a few hours later in checking the OMO email.

2.6. Afterwards

Due to the late and/or nonexistent publicity issue, only 153 teams participated this time, with a total of 436 students. Predictably the scores are lower than last year, with top scores of 30, 29, 26.

I then post all the problems on the forums, as is customary. This is now done by a Python script, so it is finished quickly.

At this point I finish up the solutions, compile the results, and post them on the website. The top seventeen teams are recognized, as usual. I breathe a sigh of relief and go to sleep.

3. General Suggestions and Guidelines

3.1. New Problem Writers

Ideally you would only write a new problem when you had an idea for one, but this kind of production schedule isn't fast enough.

I suggest, though, keeping a list (digital or otherwise; I use Workflowy) of problem ideas that you have. Any time you have an idea, just record it to the list. Then when, you have time, go back to it and see if you can construct problems out of your musings.

In that vein, you should probably be keeping your eyes open for new ideas basically all the time. Probably you are already doing this to some extent, but it helps to deliberately remember to do so.

Actually sitting down trying to come up with a new idea for a problem is a lot like doing math; in particular, it's pretty hard. I don't have any real advice on how to approach this.

Here are some points on problem design that are more specific to the OMO.

1. Probably the most important facet of an OMO problem is that it should be non-standard and nice. Things such as separating scores are strictly secondary.

2. Guess-ability is a lot more okay, because teams have enough time to at least try and prove their guesses. (On say, the NIMO, one can just guess, see the answer is correct, and then ignore the problem.)

3. Problems should be submitted on KARL (`http://karl.internetolympiad.org`).

4. I will be looking at basically every single problem, so I will try and polish ideas and make problem statements look more natural and such. I just need something to work with.

5. **Be proud of your problems!** We put a very human face on our problems by prominently featuring the author name. This is your chance to shine!

Good luck!

3.2. New Dictators

The #1 rule here is to **trust your own judgement**. Do not wait around or nothing will get done. Instead, be aggressive with giving instructions, and don't be too shy to give orders. You are leading a team here of some of the brightest, most dedicated students in the nation. They will be happy to help so long as you make it easy for them to help.[1] So make it easy to help by being clear about what needs to be done.

As a dictator you should also try very hard to set a good example. During the problem-writing phase, you should propose problems. People will take your instructions far more seriously once they see others, including yourself, following the instructions.

[1] I remember Moor Xu commenting to me that he proposed his problems to the Stanford Math Tournament rather than the Berkeley Math Tournament because Stanford made it easier to help.

Moreover, try to be early about things. Any time you write "we can discuss X later" gives X a nontrivial chance of never actually being discussed. It's fine to have a big queue of things that needs to be done as long as they're well-defined; big queues tend to scare people into actually doing stuff anyways.

A. Spring Coordination

By the way, I would just like to thank right now Thomas F. Sturm for the `csvsimple` package. You are a good man.

PR	Soln Writer	Checks	Evan	Michael	Sammy	Yang	Ray
1	Evan	4	75		75	75	75
2	Evan	5	1007	1007	1007	1007	1007
3	Evan	5	2015	2015	2015	2015	2015
4	Evan	5	2014	2014	2014	2014	2014
5	Evan	4	814	814	814	814	
6	Evan	2	46800		46800		
7	Evan	2	84		84		
8	Evan	3	2	2	2		
9	Evan	2	131072			131072	
10	Evan	4	2	2	2		2
11	Michael	3	90		90		90
12	Yang	4	954		954	954	954
13	Evan	3	2047			2047	2047
14	Evan	3	1186		1186		1186
15	Evan	3	147		147		147
16	Evan	2	119				119
17	Evan	2	343	343			
18	Evan	2	25	25			
19	Michael	2	100				100
20	Evan	2	47	47			
21	Evan	1	15				
22	Evan	2	620			620	
23	Michael	2		16909		16909	
24	Evan	5	21	21	21	21	21
25	Michael	3	9901	9901		9901	
26	Evan	1	720				
27	Michael	2	301	301			
28	Sammy	1			7800		
29	Sammy	2	15000		15000		
30	Yang	2	76			76	

B. Problems and Internal Comments

Problems are classified into five categories, $d \in \{10, 20, 30, 40, 50\}$. This was based on the old 50-problem exams, but we still retain this division of difficulty.

1. In English class, you have discovered a mysterious phenomenon – if you spend n hours on an essay, your score on the essay will be $100\left(1-4^{-n}\right)$ points if $2n$ is an integer, and 0 otherwise. For example, if you spend 30 minutes on an essay you will get a score of 50, but if you spend 35 minutes on the essay you somehow do not earn any points.

   It is 4AM, your English class starts at 8:05AM the same day, and you have four essays due at the start of class. If you can only work on one essay at a time, what is the maximum possible average of your essay scores?

2. Consider two circles of radius one, and let O and O' denote their centers. Point M is selected on either circle. If $OO' = 2014$, what is the largest possible area of triangle OMO'?

   ```
   This can be like #2 or so.
   ```

3. Suppose that m and n are relatively prime positive integers with $A = \frac{m}{n}$, where

   $$A = \frac{2+4+6+\dots+2014}{1+3+5+\dots+2013} - \frac{1+3+5+\dots+2013}{2+4+6+\dots+2014}.$$

   Find m. In other words, find the numerator of A when A is written as a fraction in simplest form.

4. The integers $1, 2, \dots, n$ are written in order on a long slip of paper. The slip is then cut into five pieces, so that each piece consists of some (nonempty) consecutive set of integers. The averages of the numbers on the five slips are 1234, 345, 128, 19, and 9.5 in some order. Compute n.

   ```
   I like this problem-it's nice for an easy one --Michael
   ```

5. Joe the teacher is bad at rounding. Because of this, he has come up with his own way to round grades, where a *grade* is a nonnegative decimal number with finitely many digits after the decimal point.

   Given a grade with digits $a_1a_2\dots a_m.b_1b_2\dots b_n$, Joe first rounds the number to the nearest 10^{-n+1}th place. He then repeats the procedure on the new number, rounding to the nearest 10^{-n+2}th, then rounding the result to the nearest 10^{-n+3}th, and so on, until he obtains an integer. For example, he rounds the number 2014.456 via $2014.456 \to 2014.46 \to 2014.5 \to 2015$.

   There exists a rational number M such that a grade x gets rounded to at least 90 if and only if $x \geq M$. If $M = \frac{p}{q}$ for relatively prime integers p and q, compute $p + q$.

```
This is inspired by a mistake that someone made at school ... –– Yang
Rephrased without limits –– Yang
```

6. Let L_n be the least common multiple of the integers $1, 2, \ldots, n$. For example, $L_{10} = 2{,}520$ and $L_{30} = 2{,}329{,}089{,}562{,}800$. Find the remainder when L_{31} is divided by 100,000.

```
This problem is pretty troll lol
I thought I would actually have to some v_p stuff until I noticed d=10
and then I was like darn
```

7. How many integers n with $10 \le n \le 500$ have the property that the hundreds digit of $17n$ and $17n + 17$ are different?

```
This is probably around #8 or so.
```

8. Let a_1, a_2, a_3, a_4, a_5 be real numbers satisfying

$$\begin{aligned}
2a_1 + a_2 + a_3 + a_4 + a_5 &= 1 + \tfrac{1}{8}a_4 \\
2a_2 + a_3 + a_4 + a_5 &= 2 + \tfrac{1}{4}a_3 \\
2a_3 + a_4 + a_5 &= 4 + \tfrac{1}{2}a_2 \\
2a_4 + a_5 &= 6 + a_1
\end{aligned}$$

Compute $a_1 + a_2 + a_3 + a_4 + a_5$.

```
This sohuld be like #10 or so.
```

9. Eighteen students participate in a team selection test with three problems, each worth up to seven points. All scores are nonnegative integers. After the competition, the results are posted by Evan in a table with 3 columns: the student's name, score, and rank (allowing ties), respectively. Here, a student's rank is one greater than the number of students with strictly higher scores (for example, if seven students score $0, 0, 7, 8, 8, 14, 21$ then their ranks would be $6, 6, 5, 3, 3, 2, 1$ respectively).

When Richard comes by to read the results, he accidentally reads the rank column as the score column and vice versa. Coincidentally, the results still made sense! If the scores of the students were $x_1 \le x_2 \le \cdots \le x_{18}$, determine the number of possible values of the 18-tuple $(x_1, x_2, \ldots, x_{18})$. In other words, determine the number of possible multisets (sets with repetition) of scores.

```
This is great. +1 –– Evan
Hmm engineer's induction is not necessarily a bad thing for d=20 ––
    Michael
```

10. Let $A_1A_2\ldots A_{4000}$ be a regular 4000-gon. Let X be the foot of the altitude from A_{1986} onto diagonal $A_{1000}A_{3000}$, and let Y be the foot of the altitude from A_{2014} onto $A_{2000}A_{4000}$. If $XY = 1$, what is the area of square $A_{500}A_{1500}A_{2500}A_{3500}$?

11. Let X be a point inside convex quadrilateral $ABCD$ with $\angle AXB + \angle CXD = 180^\circ$. If $AX = 14$, $BX = 11$, $CX = 5$, $DX = 10$, and $AB = CD$, find the sum of the areas of $\triangle AXB$ and $\triangle CXD$.

12. The points A, B, C, D, E lie on a line ℓ in this order. Suppose T is a point not on ℓ such that $\angle BTC = \angle DTE$, and $\overline{AT}$ is tangent to the circumcircle of triangle BTE. If $AB = 2$, $BC = 36$, and $CD = 15$, compute DE.

13. Suppose that g and h are polynomials of degree 10 with integer coefficients such that $g(2) < h(2)$ and

$$g(x)h(x) = \sum_{k=0}^{10} \left(\binom{k+11}{k} x^{20-k} - \binom{21-k}{11} x^{k-1} + \binom{21}{11} x^{k-1} \right)$$

holds for all nonzero real numbers x. Find $g(2)$.

```
wait lol you can replace $S(f(x))$ with $f(1)$ ;) -- Evan
OK, I just set $n=222$ to ensure $g(x)$ is irreducible (cyclotomic
    polynomial of the prime 223). Actually I just picked $223$ as the
    prime by typing "random prime number" into Wolfram Alpha, so if
    you can think of a "rounder" $p-1$ I'm all for it :) -- Evan
Hmmm, let's just use Fermat Primes for all primes in this contest. $p
    = 2^{16}+1$.
Fixed a bunch of dumb typos that I had made earlier.
If we want it to be less guessable, then maybe it would be a good idea
     to ask for $h(1)$ instead of $g(1)$ (since this also telescopes
    by hockey stick, and it could have a simple form if $n$ is small (
    like $9,12,$ or $16$)). What do you guys think about this possible
     adjustment? -- Michael
As I said below if they see that $g(1)h(1) = (n+1) \binom{2n+1}{n+1}$,
     then its pretty natural to just guess that $g(1) = n+1, h(1) = \
    binom{2n+1}{n+1}$. So asking for $h(1)$ wouldn't help against this
    . I was thinking that we should change it to $n = 10$, and simply
    just ask for $g(2)$, since I don't think that $g(2) h(2)$ is that
    easy to evaluate. But $g(2)$ is easy by geometric series, so this
    wouldn't be a bad change. --Yang
```

14. Let ABC be a triangle with incenter I and $AB = 1400$, $AC = 1800$, $BC = 2014$. The circle centered at I passing through A intersects line BC at two points X and Y. Compute the length XY.

15. In Prime Land, there are seven major cities, labelled C_0, C_1, ..., C_6. For convenience, we let $C_{n+7} = C_n$ for each $n = 0, 1, \ldots, 6$; i.e. we take the indices modulo 7. Al initially starts at city C_0.

    Each minute for ten minutes, Al flips a fair coin. If the coin land heads, and he is at city C_k, he moves to city C_{2k}; otherwise he moves to city C_{2k+1}. If the probability that Al is back at city C_0 after 10 moves is $\frac{m}{1024}$, find m.

16. Say a positive integer n is *radioactive* if one of its prime factors is strictly greater than $\sqrt{n}$. For example, $2012 = 2^2 \cdot 503$, $2013 = 3 \cdot 11 \cdot 61$ and $2014 = 2 \cdot 19 \cdot 53$ are all radioactive, but $2015 = 5 \cdot 13 \cdot 31$ is not. How many radioactive numbers have all prime factors less than 30?

17. Let $AXYBZ$ be a convex pentagon inscribed in a circle with diameter $\overline{AB}$. The tangent to the circle at Y intersects lines BX and BZ at L and K, respectively. Suppose that $\overline{AY}$ bisects $\angle LAZ$ and $AY = YZ$. If the minimum possible value of

$$\frac{AK}{AX} + \left(\frac{AL}{AB}\right)^2$$

can be written as $\frac{m}{n} + \sqrt{k}$, where m, n and k are positive integers with $\gcd(m, n) = 1$, compute $m + 10n + 100k$.

18. Find the number of pairs (m, n) of integers with $-2014 \le m, n \le 2014$ such that $x^3 + y^3 = m + 3nxy$ has infinitely many integer solutions (x, y).

19. Find the sum of all positive integers n such that $\tau(n)^2 = 2n$, where $\tau(n)$ is the number of positive integers dividing n.

20. Let ABC be an acute triangle with circumcenter O, and select E on $\overline{AC}$ and F on $\overline{AB}$ so that $\overline{BE} \perp \overline{AC}$, $\overline{CF} \perp \overline{AB}$. Suppose $\angle EOF - \angle A = 90^\circ$ and $\angle AOB - \angle B = 30^\circ$. If the maximum possible measure of $\angle C$ is $\frac{m}{n} \cdot 180^\circ$ for some positive integers m and n with $m < n$ and $\gcd(m, n) = 1$, compute $m + n$.

```
Hard for a d=40. Maybe early d=50.
```

21. Let $b = \frac{1}{2}(-1 + 3\sqrt{5})$. Determine the number of rational numbers which can be written in the form

$$a_{2014}b^{2014} + a_{2013}b^{2013} + \cdots + a_1 b + a_0$$

where $a_0, a_1, \ldots, a_{2014}$ are nonnegative integers less than b.

22. Let $f(x)$ be a polynomial with integer coefficients such that $f(15)f(21)f(35) - 10$ is divisible by 105. Given $f(-34) = 2014$ and $f(0) \ge 0$, find the smallest possible value of $f(0)$.

```
sidenote:
idk if this problem is actually a good "numberification", but the
    point is I wanted to use the fact that $f(p)f(q)=f(p+q)f(0)\pmod{
    pq}$, and generalizations for multiple primes.

Edited and +1. Your earlier version claimed $r^2 \equiv 4 \pmod{105}$
    implies $r \equiv \pm 2$. BTW, I think this is hard for a d=30.
    -- Evan
This is much more well hidden now, nice. With the new edit, it's
    definitely harder-maybe a hard 30 or easy 40? -- Michael
My pleasure :) I'd say even mid-40 is fine. -- Evan
```

23. Let Γ_1 and Γ_2 be circles in the plane with centers O_1 and O_2 and radii 13 and 10, respectively. Assume $O_1O_2 = 2$. Fix a circle Ω with radius 2, internally tangent to Γ_1 at P and externally tangent to Γ_2 at Q . Let ω be a second variable circle internally tangent to Γ_1 at X and externally tangent to Γ_2 at Y. Line PQ meets Γ_2 again at R, line XY meets Γ_2 again at Z, and lines PZ and XR meet at M.

As ω varies, the locus of point M encloses a region of area $\frac{p}{q}\pi$, where p and q are relatively prime positive integers. Compute $p + q$.

```
This seems hard D: I can't even tell why the homothety implies a
    circle. -- Evan
I added some details to the solution- this is probably too hard for a
    30 -- Michael
Yeah okay bumped up to d=40, although even then I guess it's probably
    at the harder end. -- Evan
```

24. Let $\mathcal{P}$ denote the set of planes in three-dimensional space with positive x, y, and z intercepts summing to one. A point (x, y, z) with $\min\{x, y, z\} > 0$ lies on exactly one plane in $\mathcal{P}$. What is the maximum possible integer value of $\left(\frac{1}{4}x^2 + 2y^2 + 16z^2\right)^{-1}$?

```
Edited. I think this can be easy side of d=40. -- Evan
Er wait the edit doesn't work... you can't maximize by [the intended
    method] here, only minimize. I think we have to specify lying on
    at most (or perhaps only?) one of the planes, and minimize. We
    also have to bound to the positive octant. Also you forgot to
    invert the constant we're multiplying by so the extraction doesn't
    work. -- Sammy
oops how did I forget to invert.  Does this work? -- Evan
Yep, I think so (fixed a typo that used to say "summing to one $1$").
    -- Sammy
```

25. If

$$\sum_{n=1}^{\infty} \frac{\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n}}{\binom{n+100}{100}} = \frac{p}{q}$$

for relatively prime positive integers p, q, find $p + q$.

```
Probably on the easy side of 50.
Maybe we can make this less guessable by shifting the sequence (i.e. $
    \sum_{n\ge1} \frac{H_{n+1}}{\binom{n+101}{100}}$). -- Evan
Wait hmm, why is this especially guessable right now? (Like even if
    you replace $100$ with smaller values, you still have to go
    through most of this process to get an answer, if I'm not mistaken
    .)--Michael
Oh okay, that's fine then. I just saw $\frac{k}{(k-1)^2}$ and was like
    oh god engineer's induction. (A bunch of our hard problems got
    trolled like this). -- Evan
```

26. Qing initially writes the ordered pair $(1, 0)$ on a blackboard. Each minute, if the pair (a, b) is on the board, she erases it and replaces it with one of the pairs $(2a - b, a)$, $(2a + b + 2, a)$ or $(a + 2b + 2, b)$. Eventually, the board reads $(2014, k)$ for some nonnegative integer k. How many possible values of k are there?

```
Easy for a d=50. We need some killer problems.
```

27. A frog starts at 0 on a number line and plays a game. On each turn the frog chooses at random to jump 1 or 2 integers to the right or left. It stops moving if it lands on a nonpositive number or a number on which it has already landed. If the expected number of times it will jump is $\frac{p}{q}$ for relatively prime positive integers p and q, find $p + q$.

```
dammit I accidentally checked this instead of the final #25. But I did
    get 301 (208/93) as well, so at least nobody needs to check this
    if we decide to use it in the future. A bit too computational for
    me though. -- Victor
```

28. In the game of Nim, players are given several piles of stones. On each turn, a player picks a nonempty pile and removes any positive integer number of stones from that pile. The player who removes the last stone wins, while the first player who cannot move loses.

Alice, Bob, and Chebyshev play a 3-player version of Nim where each player wants to win but avoids losing at all costs (there is always a player who neither wins nor loses). Initially, the piles have sizes 43, 99, x, y, where x and y are positive integers. Assuming that the first player loses when all players play optimally, compute the maximum possible value of xy.

```
This is a simple application of a result I found doing a research
    project last year (which turns out to be obscurely known).
Yeah d=40 might actually be closer. The solution is not as hard to
    think of as the long write-up makes it seem -- Sammy
Wait nvm maybe d=50 actually is closer? idk, can't tell... someone
    else should check maybe
```

29. Let $ABCD$ be a tetrahedron whose six side lengths are all integers, and let N denote the sum of these side lengths. There exists a point P inside $ABCD$ such that the feet from P onto the faces of the tetrahedron are the orthocenter of $\triangle ABC$, centroid of $\triangle BCD$, circumcenter of $\triangle CDA$, and orthocenter of $\triangle DAB$. If $CD = 3$ and $N < 100{,}000$, determine the maximum possible value of N.

```
Ok, I think it at least works now :P -Sammy
```

30. For a positive integer n, an *n-branch* B is an ordered tuple $(S_1, S_2, \dots, S_m)$ of nonempty sets (where m is any positive integer) satisfying $S_1 \subset S_2 \subset \cdots \subset S_m \subseteq \{1, 2, \dots, n\}$. An integer x is said to *appear* in B if it is an element of the last set S_m. Define an *n-plant* to be an (unordered) set of n-branches $\{B_1, B_2, \dots, B_k\}$, and call it *perfect* if each of 1, 2, $\dots$, n appears in exactly one of its branches.

Let T_n be the number of distinct perfect n-plants (where $T_0 = 1$), and suppose that for some positive real number x we have the convergence

$$\ln\left(\sum_{n\geq 0} T_n \cdot \frac{(\ln x)^n}{n!}\right) = \frac{6}{29}.$$

If $x = \frac{m}{n}$ for relatively prime positive integers m and n, compute $m + n$.

```
Can someone read and check that it's understandable? Also, $P_0 = 1,
    P_1 = 1, P_2 = 4, P_3 = 23$, if you want to check. There probably
    is a better way to word the statement.
Edited to make much more impossible to guess. -- Evan
```

C. Contest Statistics

statistics.txt

```
================================================================================
=                    Spring Contest 2014: Online Math Open                    =
=                          POST-CONTEST STATISTICS                            =
================================================================================


                         ==============================
                         =    GENERAL STATISTICS      =
                         ==============================
                            Total Teams: 153*
                            Total Students: 436

                            Average Score: 12.11
                            Median Score: 12
                            Standard Dev: 6.89


                         ==============================
                         =  DIFFICULTY OF PROBLEMS    =
                         ==============================

  By problem number           By difficulty               By accuracy
   1. 133 / 147 = 90.48%       2. 138 / 150 = 92.00%       2. 138 / 150 = 92.00%
   2. 138 / 150 = 92.00%       1. 133 / 147 = 90.48%       1. 133 / 147 = 90.48%
   3. 130 / 148 = 87.84%       3. 130 / 148 = 87.84%       3. 130 / 148 = 87.84%
   4. 118 / 137 = 86.13%       6. 122 / 143 = 85.31%       8. 106 / 123 = 86.18%
   5. 101 / 127 = 79.53%       4. 118 / 137 = 86.13%       4. 118 / 137 = 86.13%
   6. 122 / 143 = 85.31%       7. 108 / 134 = 80.60%       6. 122 / 143 = 85.31%
   7. 108 / 134 = 80.60%       8. 106 / 123 = 86.18%      10.  91 / 108 = 84.26%
   8. 106 / 123 = 86.18%       5. 101 / 127 = 79.53%       7. 108 / 134 = 80.60%
   9.  51 / 104 = 49.04%      11.  95 / 121 = 78.51%      14.  82 / 103 = 79.61%
  10.  91 / 108 = 84.26%      10.  91 / 108 = 84.26%       5. 101 / 127 = 79.53%
  11.  95 / 121 = 78.51%      16.  87 / 114 = 76.32%      11.  95 / 121 = 78.51%
  12.  39 /  92 = 42.39%      14.  82 / 103 = 79.61%      16.  87 / 114 = 76.32%
  13.  56 /  87 = 64.37%      15.  76 / 106 = 71.70%      15.  76 / 106 = 71.70%
  14.  82 / 103 = 79.61%      19.  73 / 109 = 66.97%      19.  73 / 109 = 66.97%
  15.  76 / 106 = 71.70%      13.  56 /  87 = 64.37%      13.  56 /  87 = 64.37%
  16.  87 / 114 = 76.32%       9.  51 / 104 = 49.04%      25.  47 /  78 = 60.26%
  17.  22 /  66 = 33.33%      25.  47 /  78 = 60.26%      18.  44 /  88 = 50.00%
  18.  44 /  88 = 50.00%      18.  44 /  88 = 50.00%       9.  51 / 104 = 49.04%
  19.  73 / 109 = 66.97%      12.  39 /  92 = 42.39%      12.  39 /  92 = 42.39%
  20.  19 /  68 = 27.94%      22.  32 /  83 = 38.55%      22.  32 /  83 = 38.55%
  21.  23 /  78 = 29.49%      21.  23 /  78 = 29.49%      17.  22 /  66 = 33.33%
  22.  32 /  83 = 38.55%      17.  22 /  66 = 33.33%      21.  23 /  78 = 29.49%
  23.  11 /  63 = 17.46%      20.  19 /  68 = 27.94%      20.  19 /  68 = 27.94%
  24.  14 /  69 = 20.29%      24.  14 /  69 = 20.29%      24.  14 /  69 = 20.29%
  25.  47 /  78 = 60.26%      26.  13 /  69 = 18.84%      26.  13 /  69 = 18.84%
  26.  13 /  69 = 18.84%      23.  11 /  63 = 17.46%      23.  11 /  63 = 17.46%
  27.  10 /  71 = 14.08%      27.  10 /  71 = 14.08%      27.  10 /  71 = 14.08%
  28.   3 /  59 =  5.08%      30.   5 /  63 =  7.94%      30.   5 /  63 =  7.94%
  29.   4 /  60 =  6.67%      29.   4 /  60 =  6.67%      29.   4 /  60 =  6.67%
  30.   5 /  63 =  7.94%      28.   3 /  59 =  5.08%      28.   3 /  59 =  5.08%


                         ==============================
                         =  DISTRIBUTION OF SCORES    =
                         ==============================

                 1.   30  100.00%   ***** ***** ***** ***** ***** *****
                 2.   29   99.35%   ***** ***** ***** ***** ***** **.**
                 3.   26   98.69%   ***** ***.* ***** ***** *.*** **..*
                 4.   25   98.04%   ***** ***** ***** *.**. ***** .*..*
                 5.   25   97.39%   ***** ***** ***** ***** .**** ....*
                 6.   25   96.73%   ***** ***** ***** ***** **.** *....
                 7.   24   96.08%   ***** ***** ***** ***.. ***** ..*..
                 8.   23   95.42%   ***** ***** ***** ***** ..*.* ...*.
                 9.   23   94.77%   ***** ***** ***** *.*** *...* **...
                10.   23   94.12%   ***** ***** ***** .***. *.*** *....
                11.   23   93.46%   ***** *.*** ***** ****. ***** .....
                12.   23   92.81%   ***** ***** ****. .**** **.** *....
                13.   23   92.16%   ***** ***** *.*** ***** **..* *....
                14.   23   91.50%   ***** ***** ***** ***** .*.** .....
                15.   22   90.85%   ***** ***** ***** ***.* **... .*...
                16.   22   90.20%   ***** ***.* ***** ****. **..* .*...
                17.   22   89.54%   ***** ***.* ***** **.*. ***.* *....
                18.   21   88.89%   ***** ***** ***** *.**. .*..* .*...
                19.   21   88.24%   ***** ***** *.*** *..*. **.** *....
                20.   21   87.58%   ***** ***** *.*** *..** **..* *....
                21.   21   86.93%   ***** ***** *.*** ****. **..* .....
                22.   20   86.27%   ***** ***** ****. *.**. .*.*. ..*..
                23.   20   85.62%   ***.* ***.* ***** *.**. *.*.* *....
                24.   20   84.97%   ***** ***** *.*** *.**. .*..* *....
```

25.	20	84.31%	*****	*****	*.***	*.**.	**..*	.....
26.	19	83.66%	*****	*****	*****	*..*.	.*.*.	.....
27.	19	83.01%	*****	*****	*.***	****.	....*	.....
28.	19	82.35%	*****	*****	*.***	*.**.	**...	.....
29.	19	81.70%	*****	*****	*.***	*.**.	.*..*	.....
30.	19	81.05%	*****	*****	*****	*.**.	....*	.....
31.	18	80.39%	*****	.*..*	.****	*.**.	.*..*	.*.*.
32.	18	79.74%	***..	**.**	*****	****.	.*..*	.....
33.	18	79.08%	*****	***.*	****.	****.	....*	.....
34.	18	78.43%	*****	*****	**.**	*..*.	.*..*	.....
35.	18	77.78%	*****	*****	*.***	*.**.	....*	.....
36.	17	77.12%	*****	*****	*.***	*..*.	....*	.....
37.	16	76.47%	****.	*****	***..	*.***	.....	.....
38.	16	75.82%	****.	*****	*..**	*..*.	*...*	.....
39.	16	75.16%	*****	*.*.*	*****	*.*..	....*	.....
40.	16	74.51%	*****	***.*	**.**	*.**.	.....	.....
41.	16	73.86%	*****	***.*	*****	*..*.	.....	.....
42.	16	73.20%	*****	.****	*.***	*..*.	....*	.....
43.	16	72.55%	*****	*****	*..**	*..*.	....*	.....
44.	16	71.90%	*****	***.*	*.***	*..*.	....*	.....
45.	15	71.24%	.****	*.*.*	**.**	****.	.....	.....
46.	15	70.59%	*****	*.***	*..*.	****.	.....	.....
47.	15	69.93%	**.**	***.*	*****	*....	....*	.....
48.	15	69.28%	*****	*****	**.**	...*.	.....	.....
49.	15	68.63%	*****	***.*	**.**	*..*.	.....	.....
50.	15	67.97%	*****	*****	*.**.	..**.	.....	.....
51.	15	67.32%	****.	***.*	*.***	*..*.	....*	.....
52.	15	66.67%	*****	*****	*.**.	*..*.	.....	.....
53.	15	66.01%	*****	*****	*..**	*..*.	.....	.....
54.	14	65.36%	*****	***.*	.....	..***	*...*	.....
55.	14	64.71%	****.	*****	*..**	...**	.....	.....
56.	14	64.05%	*****	***.*	*..**	...**	.....	.....
57.	14	63.40%	*****	***.*	*..*.	*..**	.....	.....
58.	14	62.75%	*****	***..	*..*.	*.**.	*....	.....
59.	14	62.09%	*****	***.*	*****	.....	.....	.....
60.	14	61.44%	.****	***.*	**.**	*..*.	.....	.....
61.	14	60.78%	*****	***.*	**.*.	*..*.	.....	.....
62.	14	60.13%	****.	*****	*.*.*	*.*..	.....	.....
63.	14	59.48%	*****	*****	*..**	..*..	.....	.....
64.	14	58.82%	*****	*.***	*.**.	*....	....*	.....
65.	14	58.17%	*****	***.*	*.***	.....	....*	.....
66.	14	57.52%	**.**	***.*	*..**	*..*.	....*	.....
67.	14	56.86%	**.**	*****	*.**.	*..*.	.....	.....
68.	14	56.21%	*****	*****	*...*	*..*.	.....	.....
69.	14	55.56%	*****	***.*	*.**.	*..*.	.....	.....
70.	13	54.90%	*.**.	*.*.*	**.*.	*.**.	.*...	.....
71.	13	54.25%	****.	***..	*..**	*.**.	.....	.....
72.	13	53.59%	*****	***.*	*...*	*....	....*	.....
73.	13	52.94%	*****	.****	*.*.*	*....	.....	.....
74.	13	52.29%	*****	*****	....*	*..*.	.....	.....
75.	13	51.63%	*****	***.*	*..**	*....	.....	.....
76.	12	50.98%	****.	*...*	**.*.	**...	.*...	.....
77.	12	50.33%	****.	.**.*	*.**.	...*.	....*	.....
78.	12	49.67%	**.**	*****	*..**	.....	.....	.....
79.	12	49.02%	*****	***.*	..**.	*....	.....	.....
80.	12	48.37%	*****	**...	*..**	*..*.	.....	.....
81.	12	47.71%	*****	***..	...**	*..*.	.....	.....
82.	12	47.06%	*****	***..	*..*.	*..*.	.....	.....
83.	12	46.41%	***.*	***.*	*..**	*....	.....	.....
84.	12	45.75%	*****	***.*	...**	*....	.....	.....
85.	11	45.10%	*****	***.*	.....	*.*..	.....	.....
86.	11	44.44%	****.	*****	....*	*....	.....	.....
87.	11	43.79%	*****	***..	*.*.*	.....	.....	.....
88.	11	43.14%	*****	*.*.*	*.**.	.....	.....	.....
89.	11	42.48%	*****	.**..	*..**	*....	.....	.....
90.	10	41.83%	*****	***..	*....	.....	.*...	.....
91.	10	41.18%	***.*	***..	....*	*..*.	.....	.....
92.	10	40.52%	*****	**..*	....*	*....	.....	.....
93.	10	39.87%	.****	***..	*...*	*....	.....	.....
94.	9	39.22%	****.	..*..	.*...	*.*.*	.....	.....
95.	9	38.56%	***.*	**...	*....	...**	.....	.....
96.	9	37.91%	*****	*.*..	.....	*....	.*...	.....
97.	9	37.25%	*..*.	***..	*...*	...*.	....*	.....
98.	9	36.60%	**.**	**.**	....*	.....	.....	.....
99.	9	35.95%	****.	*.*.*	*....	*....	.....	.....
100.	9	35.29%	.****	***..	*....	*....	.....	.....
101.	9	34.64%	*****	**..*	*....	.....	.....	.....
102.	9	33.99%	*****	***.*	.....	.....	.....	.....
103.	9	33.33%	*****	***..	*....	.....	.....	.....
103.	9	33.33%	*****	***..	*....	.....	.....	.....
105.	8	32.03%	****.	.**..	.....	*....	.....	.*...
106.	8	31.37%	***.*	*..*.	*...*	.....	.....	.....
107.	8	30.72%	****.	***..	.....	...*.	.....	.....
108.	8	30.07%	***..	*.*.*	*..*.	.....	.....	.....
109.	8	29.41%	**.**	**..*	.....	*....	.....	.....
110.	8	28.76%	*****	**...	.....	*....	.....	.....
111.	8	28.10%	****.	*.*.*	*....	.....	.....	.....
112.	8	27.45%	*****	.**..	*....	.....	.....	.....
113.	8	26.80%	*****	***..	.....	.....	.....	.....
114.	7	26.14%	*****	*....	.....	.....	.*...	.....
115.	7	25.49%	****.	.*...	....*	...*.	.....	.....
116.	7	24.84%	***.*	***..	.....	.....	.....	.....
117.	6	24.18%	****.	.....	..*..	*....	.....	.....
118.	6	23.53%	**...	.**..	....*	...*.	.....	.....
119.	6	22.88%	.***..	*....	*..*.	*....	.....	.....
120.	6	22.22%	**.**	**...	.....	.....	.....	.....
121.	5	21.57%	.*...	*.*..	...*.	.....	.*...	.....

```
      122.   5   20.92%   ***.*  *....  .....  .....  .....  .....
      123.   5   20.26%   .***.  **...  .....  .....  .....  .....
      124.   5   19.61%   ***..  **...  .....  .....  .....  .....
      125.   5   18.95%   ****.  *....  .....  .....  .....  .....
      126.   4   18.30%   ..**.  .*..*  .....  .....  .....  .....
      127.   4   17.65%   **.**  .....  .....  .....  .....  .....
      128.   4   16.99%   *.*.*  *....  .....  .....  .....  .....
      129.   4   16.34%   *.*..  **...  .....  .....  .....  .....
      130.   4   15.69%   *.**.  *....  .....  .....  .....  .....
      131.   3   15.03%   ..*..  *.*..  .....  .....  .....  .....
      132.   3   14.38%   *.**.  .....  .....  .....  .....  .....
      133.   3   13.73%   **.*.  .....  .....  .....  .....  .....
      134.   3   13.07%   .**..  *....  .....  .....  .....  .....
      134.   3   13.07%   .**..  *....  .....  .....  .....  .....
      136.   3   11.76%   **...  *....  .....  .....  .....  .....
      137.   3   11.11%   ***..  .....  .....  .....  .....  .....
      137.   3   11.11%   ***..  .....  .....  .....  .....  .....
      137.   3   11.11%   ***..  .....  .....  .....  .....  .....
      140.   2    9.15%   *....  .*...  .....  .....  .....  .....
      141.   2    8.50%   ..*..  *....  .....  .....  .....  .....
      142.   2    7.84%   .**..  .....  .....  .....  .....  .....
      143.   2    7.19%   **...  .....  .....  .....  .....  .....
      143.   2    7.19%   **...  .....  .....  .....  .....  .....
      143.   2    7.19%   **...  .....  .....  .....  .....  .....
      146.   1    5.23%   .....  .....  .....  .....  ....*  .....
      147.   1    4.58%   ...*.  .....  .....  .....  .....  .....
      148.   1    3.92%   ..*..  .....  .....  .....  .....  .....
      148.   1    3.92%   ..*..  .....  .....  .....  .....  .....
      150.   1    2.61%   *....  .....  .....  .....  .....  .....
      151.   1    1.96%   .*...  .....  .....  .....  .....  .....
      151.   1    1.96%   .*...  .....  .....  .....  .....  .....
      151.   1    1.96%   .*...  .....  .....  .....  .....  .....


* The team count includes only teams with positive scores.
  Hence, the number of submissions may exceed the number of teams.
  Last updated Sat Apr 19 08:51:09 2014
```