# Egregiously Ludicrous Scripted Mathematical Obfuscation

Year: **2016**

**18th ELSMO**
Pittsburgh, PA

Day: **1**

*Saturday, June 18, 2016*
*1:15PM — 5:45PM*

**Problem 1.** *CookieMonstersaysapositiveinteger* n *is* **crunchy** *ifthereexist* 2n *realnumbers* x_1,x_2,...,x_2n , *notallequal, suchthatthesumofany* n *ofthe* x_i *'sisequaltotheproductoftheother* n *ofthe* x_i *'s.HelpCookieMonsterdetermineall* **crunchy** *integers.*

**Problem 2.** *Oscarisdrawingdiagramswithtrashcanlidsandsticks.Hedrawsatriangle* ABC *andapoint* D *suchthat* DB *and* DC *aretangenttothecircumcircleof* ABC *.Let* B' *bethereflectionof* B *over* AC *and* C' *bethereflectionof* C *over* AB *.If* O *isthecircumcenterof* DB'C' *,helpOscarprovethat* AO *isperpendicularto* BC .

**Problem 3.** *InaCartesiancoordinateplane, callarectangle* **standard** *ifallofitssidesareparalleltothe* x *−and* y *−axes, andcallasetofpoints* **nice** *ifnotwoofthemhavethesame* x *−or* y *−coordinates. First, Bertchoosesa* **nice** *set* B *of* 2016 *pointsinthecoordinateplane.TomesswithBert, Ernie thenchoosesaset* E *of* n *pointsinthecoordinateplane suchthat* B \cup E *isaniceset with* 2016+n *points. Bertreturnsandthenmiraculouslynoticesthattheredoesnotexista* **standard** *rectanglethatcontainsatleasttwopointsin* B *andnopointsin* E *initsinterior.Foragiven* **nice** *set* B *thatBertchooses, define* f(B) *asthesmallestpositiveinteger* n *suchthatErniecanfinda* **nice** *set* E *ofsize* n *withtheaforementionedproperties. HelpBertdeterminetheminimumandmaximumpossiblevaluesof* f(B) .

*Time limit: 4 hours 30 minutes.*
*Each problem is worth 7 points.*

# Egregiously Ludicrous Scripted Mathematical Obfuscation

Year: **2016**

**18th ELSMO**
Pittsburgh, PA

Day: **2**

*Sunday, June 19, 2016*
*1:15PM — 5:45PM*

**Problem 4.** *BigBirdhasapolynomial* P *withintegercoefficientssuchthat* n *divides* P( $2^n$ ) *foreverypositiveinteger* n *.ProvethatBigBird'spolynomialmustbethezeropolynomial.*

**Problem 5.** *Elmoisdrawingwithcoloredchalkonasidewalkoutside.Hefirstmarksaset* S *of* n > 1 *collinearpoints.Then, foreveryunorderedpairofpoints* {X,Y} *in* S , *Elmodrawsthecirclewithdiameter* XY *sothateachpairofcircleswhichintersectattwodistinctpoints aredrawnindifferentcolors.CountvonCountthenwishestocountthenumberofcolorsElmoused. Intermsof* n *, whatistheminimumnumberofcolorsElmocouldhaveused?*

**Problem 6.** *Elmoisnowlearningolympiadgeometry.Inatriangle* ABC *with* AB ≠ AC *, letitsincirclebetangenttosides* BC , CA *, and* AB *at* D , E *, and* F *, respectively. Theinternalanglebisectorof* \angle BAC *intersectslines* DE *and* DF *at* X *and* Y *, respectively.Let* S *and* T *bedistinctpointsonside* BC *suchthat* \angle XSY=\angle XTY=90 ° . *Finally, let* \gamma *bethecircumcircleof* \triangle AST .
(a) *HelpElmoshowthat* \gamma *istangenttothecircumcircleof* \triangle ABC.
(b) *HelpElmoshowthat* \gamma *isalsotangenttotheincircleof* \triangle ABC.

*Time limit: 4 hours 30 minutes.*
*Each problem is worth 7 points.*