Exceedingly Luck-based Math Olympiad
Day 1

1. Determine all (not necessarily finite) sets S of points in the plane such that given any four distinct points in S, there is a circle passing through all four or a line passing through some three.

2. Let r and s be positive integers. Define $a_0 = 0$, $a_1 = 1$, and $a_n = ra_{n-1} + sa_{n-2}$ for $n \geq 2$. Let $f_n = a_1 a_2 \cdots a_n$. Prove that $\dfrac{f_n}{f_k f_{n-k}}$ is an integer for all integers n and k such that $0 < k < n$.

3. Let $n > 1$ be a positive integer. A 2-dimensional grid, infinite in all directions, is given. Each 1 by 1 square in a given n by n square has a counter on it. A *move* consists of taking n adjacent counters in a row or column and sliding them each by one space along that row or column. A *returning sequence* is a finite sequence of moves such that all counters again fill the original n by n square at the end of the sequence.

   (a) Assume that all counters are distinguishable except two, which are indistinguishable from each other. Prove that any distinguishable arrangement of counters in the n by n square can be reached by a returning sequence.

   (b) Assume all counters are distinguishable. Prove that there is no returning sequence that switches two counters and returns the rest to their original positions.

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Day 2

4. Determine all strictly increasing functions $f : \mathbb{N} \to \mathbb{N}$ satisfying $nf(f(n)) = f(n)^2$ for all positive integers n.

5. 2010 MOPpers are assigned numbers 1 through 2010. Each one is given a red slip and a blue slip of paper. Two positive integers, A and B, each less than or equal to 2010 are chosen. On the red slip of paper, each MOPper writes the remainder when the product of A and his or her number is divided by 2011. On the blue slip of paper, he or she writes the remainder when the product of B and his or her number is divided by 2011. The MOPpers may then perform either of the following two operations:

   - Each MOPper gives his or her red slip to the MOPper whose number is written on his or her blue slip.
   - Each MOPper gives his or her blue slip to the MOPper whose number is written on his or her red slip.

   Show that it is always possible to perform some number of these operations such that each MOPper is holding a red slip with his or her number written on it.

6. Let ABC be a triangle with circumcircle ω, incenter I, and A-excenter I_A. Let the incircle and the A-excircle hit BC at D and E, respectively, and let M be the midpoint of arc BC without A. Consider the circle tangent to BC at D and arc BAC at T. If TI intersects ω again at S, prove that SI_A and ME meet on ω.